Triangle Congruence: Diagrams, CPCTC and Proofs

SSS, SAS, ASA, AAS and HL from the diagram, then two-column proofs

Name _____ Date _____ Score _____ / 29

PART A Name the postulate

Write SSS, SAS, ASA, AAS or HL. If the given parts do not force the triangles to be congruent, write **NEI** (not enough information).

1. $\overline{AB} \cong \overline{DE},\ \overline{BC} \cong \overline{EF},\ \overline{AC} \cong \overline{DF}$ _____

2. $\overline{PQ} \cong \overline{ST},\ \angle Q \cong \angle T,\ \overline{QR} \cong \overline{TU}$ _____

3. $\angle J \cong \angle M,\ \overline{JK} \cong \overline{MN},\ \angle K \cong \angle N$ _____

4. $\angle G \cong \angle R,\ \angle H \cong \angle S,\ \overline{GI} \cong \overline{RT}$ _____

5. $\overline{AB} \cong \overline{DE},\ \overline{BC} \cong \overline{EF},\ \angle A \cong \angle D$ _____

6. $\angle X \cong \angle A,\ \angle Y \cong \angle B,\ \angle Z \cong \angle C$ _____

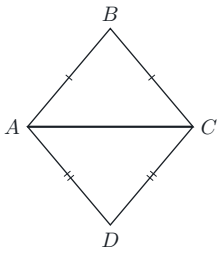
7. $\angle B$ and $\angle E$ are right angles, $\overline{AC} \cong \overline{DF}$ (hypotenuses), $\overline{BC} \cong \overline{EF}$ (legs) _____

8. $\overline{LM} \cong \overline{WX},\ \angle M \cong \angle X,\ \angle N \cong \angle Y$ _____

PART B Read the diagram

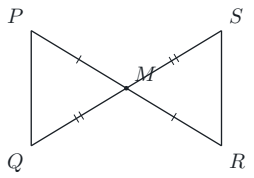
Use the marks in each figure. Matching tick marks mean congruent segments; matching arrowheads mean parallel lines.

9. $\overline{AC}$ is shared by both triangles.



Postulate: _____

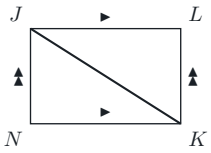
10. $\angle PMQ$ and $\angle RMS$ are vertical angles.



Postulate: _____

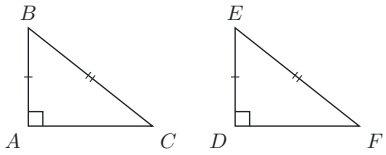
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11. $\overline{JL} \parallel \overline{NK}$ and $\overline{JN} \parallel \overline{LK}$.



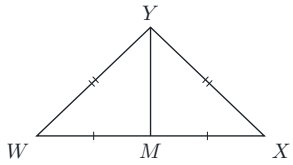
Postulate: _____

12. $\angle A$ and $\angle D$ are right angles.



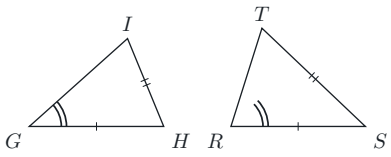
Postulate: _____

13. M is the midpoint of $\overline{WX}$ and $\overline{YM}$ is shared.



Postulate: _____

14. The marked angles are not between the two marked sides.



Postulate: _____

PART C CPCTC

Corresponding Parts of Congruent Triangles are Congruent. Once two triangles are congruent, every matching pair of parts is congruent.

15. $\triangle ABC \cong \triangle DEF$. Name the side congruent to $\overline{BC}$. _____

16. $\triangle PQR \cong \triangle XYZ$. Name the angle congruent to $\angle R$. _____

17. $\triangle LMN \cong \triangle STU$. Name the side congruent to $\overline{LN}$. _____

18. $\triangle GHI \cong \triangle JKL$ and $GH = 7$ cm. Find JK. _____

PART D Two-column proofs

Fill in the missing statements and reasons.

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Use the figure from question 13.

Given: M is the midpoint of $\overline{WX}$; $\overline{WY} \cong \overline{XY}$ **Prove:** $\triangle WMY \cong \triangle XMY$

Statements	**Reasons**
M is the midpoint of $\overline{WX}$	Given
_____	Definition of midpoint
$\overline{WY} \cong \overline{XY}$	Given
$\overline{YM} \cong \overline{YM}$	_____
$\triangle WMY \cong \triangle XMY$	_____

Use the figure from question 10.

Given: $\overline{PM} \cong \overline{RM}$; $\overline{QM} \cong \overline{SM}$ **Prove:** $\triangle PMQ \cong \triangle RMS$

Statements	**Reasons**
$\overline{PM} \cong \overline{RM}$	Given
_____	Vertical angles are congruent
$\overline{QM} \cong \overline{SM}$	Given
$\triangle PMQ \cong \triangle RMS$	_____

Use the figure from question 11.

Given: $\overline{JL} \parallel \overline{NK}$; $\overline{JN} \parallel \overline{LK}$ **Prove:** $\triangle JLK \cong \triangle KNJ$

Statements	**Reasons**
$\overline{JL} \parallel \overline{NK}$, $\overline{JN} \parallel \overline{LK}$	Given
$\angle LJK \cong \angle NKJ$	_____
_____	Reflexive property
$\angle JKL \cong \angle KJN$	Alternate interior angles are congruent
$\triangle JLK \cong \triangle KNJ$	_____

Use the figure from question 12.

Given: $\angle A$ and $\angle D$ are right angles; $\overline{BC} \cong \overline{EF}$; $\overline{AB} \cong \overline{DE}$ **Prove:** $\triangle ABC \cong \triangle DEF$

Statements	**Reasons**
$\angle A$ and $\angle D$ are right angles	Given
$\triangle ABC$ and $\triangle DEF$ are right triangles	_____
$\overline{BC} \cong \overline{EF}$ (hypotenuses)	Given
_____	Given
$\triangle ABC \cong \triangle DEF$	_____

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Answer Key

Triangle Congruence: Diagrams, CPCTC and Proofs

PART A Name the postulate

1. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$ **SSS** (all three pairs of sides)
2. $\overline{PQ} \cong \overline{ST}$, $\angle Q \cong \angle T$, $\overline{QR} \cong \overline{TU}$ **SAS** (the angle lies between the two sides)
3. $\angle J \cong \angle M$, $\overline{JK} \cong \overline{MN}$, $\angle K \cong \angle N$ **ASA** (the side lies between the two angles)
4. $\angle G \cong \angle R$, $\angle H \cong \angle S$, $\overline{GI} \cong \overline{RT}$ **AAS** (the side is not between the two angles)
5. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\angle A \cong \angle D$ **NEI** (these parts do not force congruence)
6. $\angle X \cong \angle A$, $\angle Y \cong \angle B$, $\angle Z \cong \angle C$ **NEI** (these parts do not force congruence)
7. $\angle B$ and $\angle E$ are right angles, $\overline{AC} \cong \overline{DF}$ (hypotenuses), $\overline{BC} \cong \overline{EF}$ (legs) **HL** (right triangles, hypotenuse and one leg)
8. $\overline{LM} \cong \overline{WX}$, $\angle M \cong \angle X$, $\angle N \cong \angle Y$ **AAS** (the side is not between the two angles)

PART B Read the diagram

9. **SSS** $\overline{AC}$ is shared by both triangles.
10. **SAS** $\angle PMQ$ and $\angle RMS$ are vertical angles.
11. **ASA** $\overline{JL} \parallel \overline{NK}$ and $\overline{JN} \parallel \overline{LK}$.
12. **HL** $\angle A$ and $\angle D$ are right angles.
13. **SSS** M is the midpoint of $\overline{WX}$ and $\overline{YM}$ is shared.
14. **NEI** The marked angles are not between the two marked sides.

PART C CPCTC

15. $\triangle ABC \cong \triangle DEF$. Name the side congruent to $\overline{BC}$. $\overline{EF}$
16. $\triangle PQR \cong \triangle XYZ$. Name the angle congruent to $\angle R$. $\angle Z$
17. $\triangle LMN \cong \triangle STU$. Name the side congruent to $\overline{LN}$. $\overline{SU}$
18. $\triangle GHI \cong \triangle JKL$ and $GH = 7$ cm. Find JK. $JK = 7$ **cm**

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PART D Two-column proofs

Completed. The filled parts are shown in bold.

Use the figure from question 13.

Given: M is the midpoint of $\overline{WX}$; $\overline{WY} \cong \overline{XY}$ **Prove:** $\triangle WMY \cong \triangle XMY$

Statements	Reasons
M is the midpoint of $\overline{WX}$	Given
$\overline{WM} \cong \overline{XM}$	Definition of midpoint
$\overline{WY} \cong \overline{XY}$	Given
$\overline{YM} \cong \overline{YM}$	**Reflexive property**
$\triangle WMY \cong \triangle XMY$	**SSS**

Use the figure from question 10.

Given: $\overline{PM} \cong \overline{RM}$; $\overline{QM} \cong \overline{SM}$ **Prove:** $\triangle PMQ \cong \triangle RMS$

Statements	Reasons
$\overline{PM} \cong \overline{RM}$	Given
$\angle PMQ \cong \angle RMS$	Vertical angles are congruent
$\overline{QM} \cong \overline{SM}$	Given
$\triangle PMQ \cong \triangle RMS$	**SAS**

Use the figure from question 11.

Given: $\overline{JL} \parallel \overline{NK}$; $\overline{JN} \parallel \overline{LK}$ **Prove:** $\triangle JLK \cong \triangle KNJ$

Statements	Reasons
$\overline{JL} \parallel \overline{NK}$, $\overline{JN} \parallel \overline{LK}$	Given
$\angle LJK \cong \angle NKJ$	**Alternate interior angles are congruent**
$\overline{JK} \cong \overline{JK}$	Reflexive property
$\angle JKL \cong \angle KJN$	Alternate interior angles are congruent
$\triangle JLK \cong \triangle KNJ$	**ASA**

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Use the figure from question 12.

Given: $\angle A$ and $\angle D$ are right angles; $\overline{BC} \cong \overline{EF}$; $\overline{AB} \cong \overline{DE}$ **Prove:** $\triangle ABC \cong \triangle DEF$

Statements	**Reasons**
$\angle A$ and $\angle D$ are right angles	Given
$\triangle ABC$ and $\triangle DEF$ are right triangles	**Definition of a right triangle**
$\overline{BC} \cong \overline{EF}$ (hypotenuses)	Given
$\overline{AB} \cong \overline{DE}$ **(legs)**	Given
$\triangle ABC \cong \triangle DEF$	**HL**

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