Solving Radical Equations Worksheet

Isolate, raise to the power, check · extraneous solutions · cube roots and rational exponents

Name _____ **Date** _____ **Score** _____ **/ 26**

THE METHOD

1. Get the radical on its own on one side. Nothing else can happen until it is alone.
2. Raise both sides to the power that cancels the root — square a square root, cube a cube root.
3. Solve what is left. It is often a linear equation, sometimes a quadratic.
4. Check every answer in the original equation. Squaring can create solutions that were never there.

If an answer makes the inside of a square root negative, or makes the two sides have opposite signs, it is *extraneous* and must be thrown away.

WORKED EXAMPLE — solve $\sqrt{x+2} = x$

Step 1. The radical is already alone.

Step 2. Square both sides: $x + 2 = x^2$

Step 3. Rearrange and factor: $x^2 - x - 2 = 0$, $(x-2)(x+1) = 0$, so $x = 2$ or $x = -1$.

Step 4. Check both. $x = 2$: $\sqrt{4} = 2$ ✓ $x = -1$: $\sqrt{1} = 1$, but the right side is -1, so it fails.

The square root sign means the *positive* root, so it can never equal a negative number. $x = \mathbf{2}$ is the only solution.

SECTION A Isolate and square

One square root each. Square both sides, then solve. Check your answers as you go.

1 $\sqrt{x} = 7$

Answer $x =$ _____

2 $\sqrt{x+8} = 9$

Answer $x =$ _____

3 $\sqrt{x-4} = 5$

Answer $x =$ _____

4 $2\sqrt{x} = 10$

Answer $x =$ _____

5 $\sqrt{3x-2} = 4$

Answer $x =$ _____

6 $\sqrt{x} + 3 = 8$

Answer $x =$ _____

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Solving Radical Equations Worksheet

Where squaring invents answers · a radical on each side

SECTION B Extraneous solutions

Squaring turns each of these into a quadratic with two roots. At least one of them is not a real solution. Check both, every time.

7 $\sqrt{x+2} = x$

Answer $x =$ _____ (check every answer)

8 $\sqrt{2x+8} = x$

Answer $x =$ _____ (check every answer)

9 $\sqrt{5-x} = x+1$

Answer $x =$ _____ (check every answer)

10 $\sqrt{x+7} = x-5$

Answer $x =$ _____ (check every answer)

11 $\sqrt{3x+4} = x$

Answer $x =$ _____ (check every answer)

12 $\sqrt{x+11} = x+5$

Answer $x =$ _____ (check every answer)

SECTION C A radical on each side

Square both sides once; the radicals disappear together. Then check that the inside of each root is not negative.

13 $\sqrt{2x+1} = \sqrt{x+5}$

Answer $x =$ _____

14 $\sqrt{3x-5} = \sqrt{x+7}$

Answer $x =$ _____

15 $\sqrt{4x+1} = \sqrt{2x+9}$

Answer $x =$ _____

16 $\sqrt{x-3} = \sqrt{2x-11}$

Answer $x =$ _____

17 $\sqrt{6x+2} = \sqrt{3x+14}$

Answer $x =$ _____

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Solving Radical Equations Worksheet

Cube roots and rational exponents · the same method, a different power

CUBE ROOTS BEHAVE DIFFERENTLY

A cube root is defined for negative numbers and returns a negative answer, so $\sqrt[3]{-8} = -2$. Cubing both sides is reversible, which is why cube root equations do not produce extraneous solutions the way square roots do. You should still check, but nothing will be thrown away.

Rational exponents are the same thing written differently: $x^{2/3}$ means $\sqrt[3]{x^2}$. To undo $x^{m/n}$, raise both sides to the power n/m.

SECTION D Cube roots

Cube both sides, then solve.

18 $\sqrt[3]{x-1} = 2$

Answer $x =$ _____

19 $\sqrt[3]{2x+3} = 3$

Answer $x =$ _____

20 $\sqrt[3]{x+5} = -2$

Answer $x =$ _____

21 $\sqrt[3]{4x} = 4$

Answer $x =$ _____

22 $\sqrt[3]{\frac{x}{2}+1} = 1$

Answer $x =$ _____

SECTION E Rational exponents

Raise both sides to the reciprocal power. Give exact answers.

23 $\sqrt{x} = 6$

Answer $x =$ _____

24 $x^{\frac{3}{2}} = 8$

Answer $x =$ _____

25 $\sqrt[3]{x} = 4$

Answer $x =$ _____

26 $x^{\frac{5}{2}} = 32$

Answer $x =$ _____

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Answer Key

Every candidate substituted back into the original equation; the ones that fail are named.

SECTION A Isolate and square

1. $\sqrt{x} = 7$

$x = 49$

2. $\sqrt{x+8} = 9$

$x = 73$

3. $\sqrt{x-4} = 5$

$x = 29$

4. $2\sqrt{x} = 10$

$x = 25$

5. $\sqrt{3x-2} = 4$

$x = 6$

6. $\sqrt{x} + 3 = 8$

$x = 25$

SECTION B Extraneous solutions

The rejected value is the one squaring invented.

7. $\sqrt{x+2} = x$

$x = 2$

$x = -1$ is extraneous — it fails the original equation

8. $\sqrt{2x+8} = x$

$x = 4$

$x = -2$ is extraneous — it fails the original equation

9. $\sqrt{5-x} = x + 1$

$x = 1$

$x = -4$ is extraneous — it fails the original equation

10. $\sqrt{x+7} = x - 5$

$x = 9$

$x = 2$ is extraneous — it fails the original equation

11. $\sqrt{3x+4} = x$

$x = 4$

$x = -1$ is extraneous — it fails the original equation

12. $\sqrt{x+11} = x + 5$

$x = -2$

$x = -7$ is extraneous — it fails the original equation

SECTION C A radical on each side

13. $\sqrt{2x+1} = \sqrt{x+5}$

$x = 4$

14. $\sqrt{3x-5} = \sqrt{x+7}$

$x = 6$

15. $\sqrt{4x+1} = \sqrt{2x+9}$

$x = 4$

16. $\sqrt{x-3} = \sqrt{2x-11}$

$x = 8$

17. $\sqrt{6x+2} = \sqrt{3x+14}$

$x = 4$

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Answer Key

Cube roots and rational exponents

SECTION D Cube roots

No extraneous solutions here.

18. $\sqrt[3]{x-1} = 2$

$x = 9$

19. $\sqrt[3]{2x+3} = 3$

$x = 12$

20. $\sqrt[3]{x+5} = -2$

$x = -13$

21. $\sqrt[3]{4x} = 4$

$x = 16$

22. $\sqrt[3]{\frac{x}{2}+1} = 1$

$x = 0$

SECTION E Rational exponents

23. $\sqrt{x} = 6$

$x = 36$

24. $x^{\frac{3}{2}} = 8$

$x = 4$

25. $\sqrt[3]{x} = 4$

$x = 64$

$x = -64$ is extraneous — it fails the original equation

26. $x^{\frac{5}{2}} = 32$

$x = 4$

Why checking is not optional: squaring both sides of $\sqrt{A} = B$ gives $A = B^2$, which is also what you get from $\sqrt{A} = -B$. The squared equation cannot tell the two apart, so it hands back answers belonging to an equation you were never solving.

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