Quadratic Formula Worksheet

Algebra 1 & 2 · discriminant, exact answers, simplest radical form, complex solutions

Name _____ **Date** _____ **Score** _____ **/ 33**

THE QUADRATIC FORMULA

For $ax^2 + bx + c = 0$ (with $a \neq 0$):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

THE DISCRIMINANT $b^2 - 4ac$

Positive perfect square → 2 rational solutions

Positive, not a perfect square → 2 irrational solutions

Zero → 1 real solution (repeated)

Negative → no real solutions (2 complex)

WORKED EXAMPLE — solve $2x^2 + 3x - 2 = 0$

Step 1. Read off $a = 2$, $b = 3$, $c = -2$. The equation must equal 0 first.

Step 2. Discriminant: $3^2 - 4(2)(-2) = 9 + 16 = 25$, a perfect square, so 2 rational solutions.

Step 3. Substitute: $x = \frac{-3 \pm \sqrt{25}}{2(2)} = \frac{-3 \pm 5}{4}$

Step 4. Split the $\pm$: $x = \frac{-3+5}{4} = \frac{1}{2}$ and $x = \frac{-3-5}{4} = -2$.

SECTION A Find a, b, c and the discriminant

Write the equation in standard form first. Then say how many real solutions it has — do not solve.

1 $x^2 + 5x + 6 = 0$

a ___ b ___ c ___

$b^2 - 4ac =$ _____

Real solutions: _____

2 $2x^2 - 4x + 2 = 0$

a ___ b ___ c ___

$b^2 - 4ac =$ _____

Real solutions: _____

3 $x^2 + 2x + 5 = 0$

a ___ b ___ c ___

$b^2 - 4ac =$ _____

Real solutions: _____

4 $3x^2 = 5x - 2$

a ___ b ___ c ___

$b^2 - 4ac =$ _____

Real solutions: _____

5 $x^2 - 7 = 0$

a ___ b ___ c ___

$b^2 - 4ac =$ _____

Real solutions: _____

6 $4x^2 + 12x + 9 = 0$

a ___ b ___ c ___

$b^2 - 4ac =$ _____

Real solutions: _____

7 $-x^2 + 3x + 4 = 0$

a ___ b ___ c ___

$b^2 - 4ac =$ _____

Real solutions: _____

8 $5x - x^2 = 1$

a ___ b ___ c ___

$b^2 - 4ac =$ _____

Real solutions: _____

Practise free online: ► Quadratic Equation Solver ► Completing the Square Worksheet ► Math Worksheets

© notesofmath.com — CC BY-NC-SA 4.0

By Abu Abdul Rahman, NotesOfMath. Free to copy, print and adapt for non-commercial educational use, with attribution — CC BY-NC-SA 4.0: creativecommons.org/licenses/by-nc-sa/4.0/

Quadratic Formula Worksheet

Rational answers

SECTION B Solve with the quadratic formula

The discriminant is a perfect square, so every answer is a whole number or a fraction. Simplify fractions.

9 $x^2 + 7x + 10 = 0$

$x =$ _____

10 $x^2 - 3x - 28 = 0$

$x =$ _____

11 $2x^2 + 5x - 3 = 0$

$x =$ _____

12 $3x^2 - 10x + 8 = 0$

$x =$ _____

13 $6x^2 + x - 2 = 0$

$x =$ _____

14 $x^2 - 6x + 9 = 0$

$x =$ _____

15 $4x^2 - 9 = 0$

$x =$ _____

16 $5x^2 + 3x = 0$

$x =$ _____

THE MISTAKES TO AVOID

1. $-b$ means the *opposite* of b: if $b = -3$, then $-b = 3$. **2.** Square negatives in brackets: $(-3)^2 = 9$, not -9. **3.** Divide the *whole* top by $2a$, not just the first term.

Practise free online: ► Quadratic Equation Solver ► Completing the Square Worksheet ► Math Worksheets

© notesofmath.com — CC BY-NC-SA 4.0

By Abu Abdul Rahman, NotesOfMath. Free to copy, print and adapt for non-commercial educational use, with attribution — CC BY-NC-SA 4.0: creativecommons.org/licenses/by-nc-sa/4.0/

Quadratic Formula Worksheet

Simplest radical form

SIMPLIFYING THE ANSWER — example $x^2 - 2x - 2 = 0$

$x = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$ Take square factors out of the root ($\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$), then divide *every* term on top by the bottom.

SECTION C Answers in simplest radical form

Simplify the square root, then divide out any common factor.

17 $x^2 + 4x + 1 = 0$

$x =$ _____

18 $x^2 - 6x + 4 = 0$

$x =$ _____

19 $2x^2 + 2x - 1 = 0$

$x =$ _____

20 $x^2 + 3x - 5 = 0$

$x =$ _____

21 $3x^2 - 8x + 2 = 0$

$x =$ _____

22 $2x^2 - 4x - 3 = 0$

$x =$ _____

© notesofmath.com — CC BY-NC-SA 4.0

By Abu Abdul Rahman, NotesOfMath. Free to copy, print and adapt for non-commercial educational use, with attribution — CC BY-NC-SA 4.0: creativecommons.org/licenses/by-nc-sa/4.0/

Quadratic Formula Worksheet

No real solutions · standard form first

SECTION D Negative discriminant

Algebra 1: write *no real solutions*. Algebra 2: also give the complex solutions using $i = \sqrt{-1}$.

23 $x^2 + 2x + 5 = 0$

$x =$ _____

24 $x^2 - 4x + 13 = 0$

$x =$ _____

25 $2x^2 + 2x + 1 = 0$

$x =$ _____

26 $x^2 + x + 1 = 0$

$x =$ _____

SECTION E Get everything on one side first

Move every term to the left so the right side is 0, then solve.

27 $x^2 = 4x + 5$

$x =$ _____

28 $2x^2 + 3 = 7x$

$x =$ _____

29 $x(x + 6) = 2$

$x =$ _____

30 $(x - 1)^2 = 3x$

$x =$ _____

© notesofmath.com — CC BY-NC-SA 4.0

By Abu Abdul Rahman, NotesOfMath. Free to copy, print and adapt for non-commercial educational use, with attribution — CC BY-NC-SA 4.0: creativecommons.org/licenses/by-nc-sa/4.0/

Quadratic Formula Worksheet

Word problems

SECTION F Word problems

Solve, then check whether each answer makes sense. A time or a length cannot be negative.

31 A ball is thrown up from an 80-ft roof. Its height after t seconds is $h = -16t^2 + 64t + 80$. When does it hit the ground ($h = 0$)?

Answer _____

32 A rectangle is 3 cm longer than it is wide, and its area is 40 cm^2. Write $w(w + 3) = 40$ and find the width w.

Answer _____

33 A rocket's height in metres after t seconds is $h = -5t^2 + 20t + 3$. When does it hit the ground? Give an exact answer and a decimal.

Answer _____

WHICH METHOD SHOULD I USE?

Factoring is fastest when the discriminant is a perfect square and the numbers are small. **Completing the square** is useful for vertex form. The **quadratic formula** works for *every* quadratic, so use it whenever factoring is not quick.

Check your answers with the free quadratic equation solver on notesofmath.com.

© notesofmath.com — CC BY-NC-SA 4.0

By Abu Abdul Rahman, NotesOfMath. Free to copy, print and adapt for non-commercial educational use, with attribution — CC BY-NC-SA 4.0: creativecommons.org/licenses/by-nc-sa/4.0/

Answer Key

Every answer worked from a, b and c and checked in the original equation

SECTION A a, b, c and the discriminant

1. $x^2 + 5x + 6 = 0$

$a = 1,\ b = 5,\ c = 6$

$5^2 - 4(1)(6) = 1$

► 2 real (rational)

2. $2x^2 - 4x + 2 = 0$

$a = 2,\ b = -4,\ c = 2$

$(-4)^2 - 4(2)(2) = 0$

► 1 real (repeated)

3. $x^2 + 2x + 5 = 0$

$a = 1,\ b = 2,\ c = 5$

$2^2 - 4(1)(5) = -16$

► 0 real (2 complex)

4. $3x^2 = 5x - 2$

$3x^2 - 5x + 2 = 0$

$a = 3,\ b = -5,\ c = 2$

$(-5)^2 - 4(3)(2) = 1$

► 2 real (rational)

5. $x^2 - 7 = 0$

$a = 1,\ b = 0,\ c = -7$

$0^2 - 4(1)(-7) = 28$

► 2 real (irrational)

6. $4x^2 + 12x + 9 = 0$

$a = 4,\ b = 12,\ c = 9$

$12^2 - 4(4)(9) = 0$

► 1 real (repeated)

7. $-x^2 + 3x + 4 = 0$

$a = -1,\ b = 3,\ c = 4$

$3^2 - 4(-1)(4) = 25$

► 2 real (rational)

8. $5x - x^2 = 1$

$-x^2 + 5x - 1 = 0$

$a = -1,\ b = 5,\ c = -1$

$5^2 - 4(-1)(-1) = 21$

► 2 real (irrational)

SECTION B Rational answers

9. $x^2 + 7x + 10 = 0$

$a = 1,\ b = 7,\ c = 10$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(10)}}{2(1)}$$

discriminant $b^2 - 4ac = 9$

$$x = \frac{-7 \pm 3}{2}$$

► $x = -2$ or $x = -5$

10. $x^2 - 3x - 28 = 0$

$a = 1,\ b = -3,\ c = -28$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-28)}}{2(1)}$$

discriminant $b^2 - 4ac = 121$

$$x = \frac{3 \pm 11}{2}$$

► $x = 7$ or $x = -4$

11. $2x^2 + 5x - 3 = 0$

$a = 2,\ b = 5,\ c = -3$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-3)}}{2(2)}$$

discriminant $b^2 - 4ac = 49$

$$x = \frac{-5 \pm 7}{4}$$

► $x = \frac{1}{2}$ or $x = -3$

12. $3x^2 - 10x + 8 = 0$

$a = 3,\ b = -10,\ c = 8$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(8)}}{2(3)}$$

discriminant $b^2 - 4ac = 4$

$$x = \frac{10 \pm 2}{6}$$

► $x = 2$ or $x = \frac{4}{3}$

13. $6x^2 + x - 2 = 0$

$a = 6,\ b = 1,\ c = -2$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(6)(-2)}}{2(6)}$$

discriminant $b^2 - 4ac = 49$

$$x = \frac{-1 \pm 7}{12}$$

► $x = \frac{1}{2}$ or $x = -\frac{2}{3}$

14. $x^2 - 6x + 9 = 0$

$a = 1,\ b = -6,\ c = 9$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$$

discriminant $b^2 - 4ac = 0$

$$x = \frac{6 \pm 0}{2}$$

► $x = 3$ (one solution)

15. $4x^2 - 9 = 0$

$a = 4,\ b = 0,\ c = -9$

$$x = \frac{0 \pm \sqrt{0^2 - 4(4)(-9)}}{2(4)}$$

discriminant $b^2 - 4ac = 144$

$$x = \frac{0 \pm 12}{8}$$

► $x = \frac{3}{2}$ or $x = -\frac{3}{2}$

16. $5x^2 + 3x = 0$

$a = 5,\ b = 3,\ c = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(5)(0)}}{2(5)}$$

discriminant $b^2 - 4ac = 9$

$$x = \frac{-3 \pm 3}{10}$$

► $x = 0$ or $x = -\frac{3}{5}$

© notesofmath.com — CC BY-NC-SA 4.0

By Abu Abdul Rahman, NotesOfMath. Free to copy, print and adapt for non-commercial educational use, with attribution — CC BY-NC-SA 4.0: creativecommons.org/licenses/by-nc-sa/4.0/

Answer Key

Simplest radical form and complex solutions

SECTION C Simplest radical form

17. $x^2 + 4x + 1 = 0$

$a = 1,\ b = 4,\ c = 1$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)}$

discriminant $b^2 - 4ac = 12$

$x = \dfrac{-4 \pm 2\sqrt{3}}{2}$

► $x = -2 \pm \sqrt{3}$

18. $x^2 - 6x + 4 = 0$

$a = 1,\ b = -6,\ c = 4$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$

discriminant $b^2 - 4ac = 20$

$x = \dfrac{6 \pm 2\sqrt{5}}{2}$

► $x = 3 \pm \sqrt{5}$

19. $2x^2 + 2x - 1 = 0$

$a = 2,\ b = 2,\ c = -1$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$

discriminant $b^2 - 4ac = 12$

$x = \dfrac{-2 \pm 2\sqrt{3}}{4}$

► $x = \dfrac{-1 \pm \sqrt{3}}{2}$

20. $x^2 + 3x - 5 = 0$

$a = 1,\ b = 3,\ c = -5$

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)}$

discriminant $b^2 - 4ac = 29$

$x = \dfrac{-3 \pm \sqrt{29}}{2}$

► $x = \dfrac{-3 \pm \sqrt{29}}{2}$

21. $3x^2 - 8x + 2 = 0$

$a = 3,\ b = -8,\ c = 2$

$x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(2)}}{2(3)}$

discriminant $b^2 - 4ac = 40$

$x = \dfrac{8 \pm 2\sqrt{10}}{6}$

► $x = \dfrac{4 \pm \sqrt{10}}{3}$

22. $2x^2 - 4x - 3 = 0$

$a = 2,\ b = -4,\ c = -3$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$

discriminant $b^2 - 4ac = 40$

$x = \dfrac{4 \pm 2\sqrt{10}}{4}$

► $x = \dfrac{2 \pm \sqrt{10}}{2}$

SECTION D Negative discriminant

23. $x^2 + 2x + 5 = 0$

$a = 1,\ b = 2,\ c = 5$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$

discriminant $b^2 - 4ac = -16$

$\sqrt{-16}$ is not real $\Rightarrow$ no real solutions

► no real solutions; $x = -1 \pm 2i$

24. $x^2 - 4x + 13 = 0$

$a = 1,\ b = -4,\ c = 13$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$

discriminant $b^2 - 4ac = -36$

$\sqrt{-36}$ is not real $\Rightarrow$ no real solutions

► no real solutions; $x = 2 \pm 3i$

25. $2x^2 + 2x + 1 = 0$

$a = 2,\ b = 2,\ c = 1$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(2)(1)}}{2(2)}$

discriminant $b^2 - 4ac = -4$

$\sqrt{-4}$ is not real $\Rightarrow$ no real solutions

► no real solutions; $x = \dfrac{-1 \pm i}{2}$

26. $x^2 + x + 1 = 0$

$a = 1,\ b = 1,\ c = 1$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

discriminant $b^2 - 4ac = -3$

$\sqrt{-3}$ is not real $\Rightarrow$ no real solutions

► no real solutions; $x = \dfrac{-1 \pm i\sqrt{3}}{2}$

Practise free online: ► Quadratic Equation Solver ► Completing the Square Worksheet ► Math Worksheets

© notesofmath.com — CC BY-NC-SA 4.0

By Abu Abdul Rahman, NotesOfMath. Free to copy, print and adapt for non-commercial educational use, with attribution — CC BY-NC-SA 4.0: creativecommons.org/licenses/by-nc-sa/4.0/

Answer Key

Standard form first and word problems

SECTION E Standard form first

27. $x^2 = 4x + 5$

$x^2 - 4x - 5 = 0$

$a = 1,\ b = -4,\ c = -5$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)}$

discriminant $b^2 - 4ac = 36$

$x = \dfrac{4 \pm 6}{2}$

► $x = 5$ or $x = -1$

28. $2x^2 + 3 = 7x$

$2x^2 - 7x + 3 = 0$

$a = 2,\ b = -7,\ c = 3$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(3)}}{2(2)}$

discriminant $b^2 - 4ac = 25$

$x = \dfrac{7 \pm 5}{4}$

► $x = 3$ or $x = \dfrac{1}{2}$

29. $x(x + 6) = 2$

$x^2 + 6x - 2 = 0$

$a = 1,\ b = 6,\ c = -2$

$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2(1)}$

discriminant $b^2 - 4ac = 44$

$x = \dfrac{-6 \pm 2\sqrt{11}}{2}$

► $x = -3 \pm \sqrt{11}$

30. $(x - 1)^2 = 3x$

$x^2 - 5x + 1 = 0$

$a = 1,\ b = -5,\ c = 1$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$

discriminant $b^2 - 4ac = 21$

$x = \dfrac{5 \pm \sqrt{21}}{2}$

► $x = \dfrac{5 \pm \sqrt{21}}{2}$

SECTION F Word problems

31. $-16t^2 + 64t + 80 = 0$

$a = -16,\ b = 64,\ c = 80$

$t = \dfrac{-64 \pm \sqrt{64^2 - 4(-16)(80)}}{2(-16)}$

discriminant $b^2 - 4ac = 9216$

$t = \dfrac{-64 \pm 96}{-32}$

► $t = 5$ or $t = -1$

reject $t = -1$ (time cannot be negative); it lands after 5 seconds.

32. $w^2 + 3w - 40 = 0$

$a = 1,\ b = 3,\ c = -40$

$w = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(-40)}}{2(1)}$

discriminant $b^2 - 4ac = 169$

$w = \dfrac{-3 \pm 13}{2}$

► $w = 5$ or $w = -8$

reject $w = -8$; width 5 cm, length 8 cm.

33. $-5t^2 + 20t + 3 = 0$

$a = -5,\ b = 20,\ c = 3$

$t = \dfrac{-20 \pm \sqrt{20^2 - 4(-5)(3)}}{2(-5)}$

discriminant $b^2 - 4ac = 460$

$t = \dfrac{-20 \pm 2\sqrt{115}}{-10}$

► $t = \dfrac{10 \pm \sqrt{115}}{5}$

reject the negative root; it lands after about 4.14 seconds.

© notesofmath.com — CC BY-NC-SA 4.0

By Abu Abdul Rahman, NotesOfMath. Free to copy, print and adapt for non-commercial educational use, with attribution — CC BY-NC-SA 4.0: creativecommons.org/licenses/by-nc-sa/4.0/