Law of Sines and Cosines Worksheet

Which law to use · missing sides and angles · the ambiguous case · area

Name _____ **Date** _____ **Score** _____ **/ 30**

THE TWO LAWS, AND WHICH ONE STARTS

Law of sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ **Law of cosines:** $a^2 = b^2 + c^2 - 2bc\cos A$

The law of sines needs a side *paired with the angle opposite it.* If you have that pair, it starts. If you do not, only the law of cosines can start.

What you are given	Case	Start with
Two angles and any side	AAS or ASA	law of sines
Two sides and an angle opposite one	SSA	law of sines — check the ambiguous case
Two sides and the angle between them	SAS	law of cosines
All three sides	SSS	law of cosines

After the law of cosines gives you one angle, the law of sines can finish the triangle.

WORKED EXAMPLE — $b = 7$, $\angle A = 52^\circ$, $c = 10$: find a

The angle sits *between* the two sides, so this is SAS and the law of sines cannot start — there is no side paired with its opposite angle.

$a^2 = b^2 + c^2 - 2bc\cos A = 7^2 + 10^2 - 2(7)(10)\cos 52^\circ$

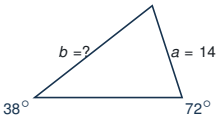
$a^2 = 49 + 100 - 140(0.6157) = 62.8$, so $a = \mathbf{7.9}$ to one decimal place.

Now $\angle B$ or $\angle C$ could be found with either law, because a is paired with $\angle A$.

SECTION A Law of sines: find a missing side

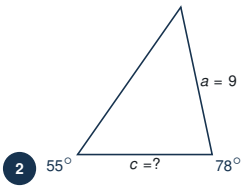
Two angles and one side. Find the third angle first, then set up one ratio. Round to one decimal place.

1



Answer $b =$ _____

2



Answer $c =$ _____

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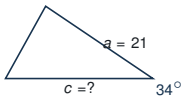

Answer $c =$ _____

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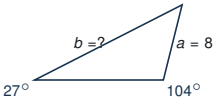

Answer $b =$ _____

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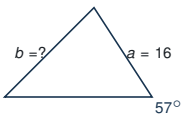

Answer $b =$ _____

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Law of Sines and Cosines Worksheet

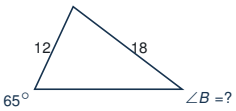
Finding angles with each law · show the ratio or the rearrangement

THE MISTAKE TO AVOID

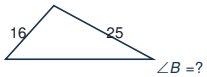
Pairing a side with the wrong angle. In every ratio the angle must be the one *opposite* that side — a with $\angle A$, never a with $\angle B$. Label the triangle before you write anything down.

SECTION B Law of sines: find a missing angle

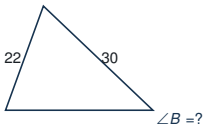
Two sides and the angle opposite one of them. In each of these the side opposite the known angle is the longer one, so there is exactly one triangle.

6 

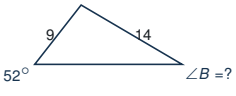
Answer $\angle B =$ _____

7 

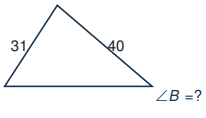
Answer $\angle B =$ _____

8 

Answer $\angle B =$ _____

9 

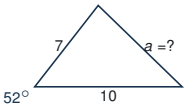
Answer $\angle B =$ _____

10 

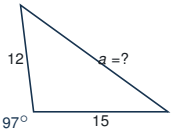
Answer $\angle B =$ _____

SECTION C Law of cosines: find a missing side

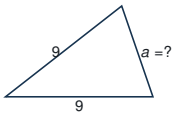
Two sides with the angle between them. Substitute into $a^2 = b^2 + c^2 - 2bc\cos A$, then square-root.

11 

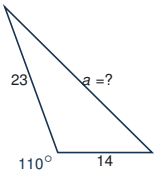
Answer $a =$ _____

12 

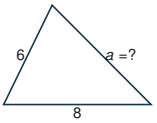
Answer $a =$ _____

13 

Answer $a =$ _____

14 

Answer $a =$ _____

15 

Answer $a =$ _____

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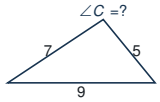
Law of Sines and Cosines Worksheet

Angles from three sides · the ambiguous case

SECTION D Law of cosines: find a missing angle

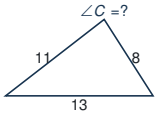
Three sides. Rearrange to $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$, then take the inverse cosine. Round to one decimal place.

16



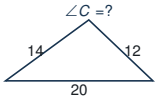
Answer $\angle C =$ _____

17



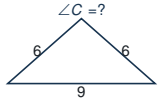
Answer $\angle C =$ _____

18



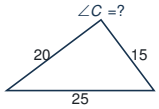
Answer $\angle C =$ _____

19



Answer $\angle C =$ _____

20



Answer $\angle C =$ _____

THE AMBIGUOUS CASE (SSA)

Your calculator's $\sin^{-1}$ only ever returns an angle between 0° and 90°, but $\sin\theta = \sin(180^\circ - \theta)$, so an obtuse angle has the same sine. In the SSA case that means a second triangle may exist. Find the acute angle, then test its obtuse partner $180^\circ - B$: if the three angles still add to less than 180°, both triangles are valid. If $\sin B$ comes out greater than 1, no triangle exists at all.

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Law of Sines and Cosines Worksheet

Naming the case, then picking the law that can start

SECTION E Which law? Then solve

For each one, name the case (AAS, ASA, SSA, SAS or SSS), say which law can start, and find the part asked for.

21 $\angle A = 42°$, $c = 19$, $\angle B = 71°$

Case _____ Law _____

Answer $a =$ _____

22 $a = 11$, $b = 13$, $c = 20$

Case _____ Law _____

Answer $\angle C =$ _____

23 $b = 8$, $\angle A = 63°$, $c = 11$

Case _____ Law _____

Answer $a =$ _____

24 $\angle B = 35°$, $\angle C = 88°$, $b = 24$

Case _____ Law _____

Answer $c =$ _____

25 $a = 9$, $b = 12$, $\angle A = 40°$

Case _____ Law _____

Answer $\angle B =$ _____

26 $a = 16$, $\angle C = 124°$, $b = 10$

Case _____ Law _____

Answer $c =$ _____

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Law of Sines and Cosines Worksheet

Area of any triangle, and two problems that start as words

AREA OF ANY TRIANGLE

Two sides and the angle between them: Area = $\frac{1}{2}ab\sin C$

Three sides (Heron's formula): $s = \dfrac{a+b+c}{2}$, Area = $\sqrt{s(s-a)(s-b)(s-c)}$

SECTION F Area and applied problems

Draw the triangle first on the word problems, and mark which parts you were given.

27 Find the area of the triangle with $a = 12$, $b = 9$ and the angle between them $\angle C = 58°$.

Answer Area = _____

28 Find the area of the triangle whose sides are 7, 8 and 9.

Answer Area = _____

29 Two observers stand 250 m apart. Each sights the same hot-air balloon: the angle at the first observer is 37° and the angle at the second is 62°. How far is the balloon from the second observer?

Answer d = _____

30 A triangular plot has two sides of 140 m and 95 m meeting at an angle of 78°. A fence is to run along the third side. How long is it?

Answer L = _____

Check a right-triangle case with the free right triangle calculator on notesofmath.com — with $\angle C = 90°$ the law of cosines becomes $c^2 = a^2 + b^2$, which is Pythagoras.

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Answer Key

Every triangle solved independently and checked against both laws.

SECTION A Law of sines: sides

Sides to one decimal place.

1. $\angle C = 70^\circ$

$b = \dfrac{14 \sin 72^\circ}{\sin 38^\circ}$

$b = \mathbf{21.6}$

2. $\angle C = 47^\circ$

$c = \dfrac{9 \sin 47^\circ}{\sin 55^\circ}$

$c = \mathbf{8}$

3. $\angle C = 85^\circ$

$c = \dfrac{21 \sin 85^\circ}{\sin 61^\circ}$

$c = \mathbf{23.9}$

4. $\angle C = 49^\circ$

$b = \dfrac{8 \sin 104^\circ}{\sin 27^\circ}$

$b = \mathbf{17.1}$

5. $\angle C = 78^\circ$

$b = \dfrac{16 \sin 57^\circ}{\sin 45^\circ}$

$b = \mathbf{19}$

SECTION B Law of sines: angles

6. $\sin B = \dfrac{12 \sin 65^\circ}{18}$

$\angle B = \mathbf{37.2^\circ}$ ($\angle C = 77.8^\circ$)

7. $\sin B = \dfrac{16 \sin 48^\circ}{25}$

$\angle B = \mathbf{28.4^\circ}$ ($\angle C = 103.6^\circ$)

8. $\sin B = \dfrac{22 \sin 70^\circ}{30}$

$\angle B = \mathbf{43.6^\circ}$ ($\angle C = 66.4^\circ$)

9. $\sin B = \dfrac{9 \sin 52^\circ}{14}$

$\angle B = \mathbf{30.4^\circ}$ ($\angle C = 97.6^\circ$)

10. $\sin B = \dfrac{31 \sin 57^\circ}{40}$

$\angle B = \mathbf{40.5^\circ}$ ($\angle C = 82.5^\circ$)

SECTION C Law of cosines: sides

11. $a^2 = 7^2 + 10^2 - 2(7)(10)\cos 52^\circ$

$a = \mathbf{7.9}$

12. $a^2 = 12^2 + 15^2 - 2(12)(15)\cos 97^\circ$

$a = \mathbf{20.3}$

13. $a^2 = 9^2 + 9^2 - 2(9)(9)\cos 38^\circ$

$a = \mathbf{5.9}$

14. $a^2 = 23^2 + 14^2 - 2(23)(14)\cos 110^\circ$

$a = \mathbf{30.7}$

15. $a^2 = 6^2 + 8^2 - 2(6)(8)\cos 64^\circ$

$a = \mathbf{7.6}$

SECTION D Law of cosines: angles

16. $\cos C = \dfrac{5^2 + 7^2 - 9^2}{2(5)(7)}$

$\angle C = \mathbf{95.7^\circ}$

17. $\cos C = \dfrac{8^2 + 11^2 - 13^2}{2(8)(11)}$

$\angle C = \mathbf{84.8^\circ}$

18. $\cos C = \dfrac{12^2 + 14^2 - 20^2}{2(12)(14)}$

$\angle C = \mathbf{100.3^\circ}$

19. $\cos C = \dfrac{6^2 + 6^2 - 9^2}{2(6)(6)}$

$\angle C = \mathbf{97.2^\circ}$

20. $\cos C = \dfrac{15^2 + 20^2 - 25^2}{2(15)(20)}$

$\angle C = \mathbf{90^\circ}$ (a right triangle)

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Answer Key

Choosing the law, the ambiguous case, area and the applied problems

SECTION E Which law? Then solve

The case decides the law: a side paired with its opposite angle means the law of sines can start.

21. ASA — law of sines

$\angle C = 67^\circ,\ a = \dfrac{19 \sin 42^\circ}{\sin 67^\circ}$

$a = \mathbf{13.8}$

22. SSS — law of cosines

$\cos C = \dfrac{11^2 + 13^2 - 20^2}{2(11)(13)}$

$\angle C = \mathbf{112.6^\circ}$

23. SAS — law of cosines

$a^2 = 8^2 + 11^2 - 2(8)(11)\cos 63^\circ$

$a = \mathbf{10.3}$

24. AAS — law of sines

$c = \dfrac{24 \sin 88^\circ}{\sin 35^\circ}$

$c = \mathbf{41.8}$

25. SSA — law of sines

$\sin B = \dfrac{12 \sin 40^\circ}{9}$ — **two triangles**

$\angle B = \mathbf{59^\circ}$ or $\mathbf{121^\circ}$

26. SAS — law of cosines

$c^2 = 16^2 + 10^2 - 2(16)(10)\cos 124^\circ$

$c = \mathbf{23.1}$

SECTION F Area and applied

27. $\frac{1}{2}(12)(9)\sin 58^\circ$

Area $= \mathbf{45.8}$ square units

28. $s = 12,\ \sqrt{12(12-7)(12-8)(12-9)}$

Area $= \mathbf{26.8}$ square units

29. $\angle C = 81^\circ,\ d = \dfrac{250 \sin 62^\circ}{\sin 37^\circ}$

$d = \mathbf{366.8}$ m

30. $L^2 = 140^2 + 95^2 - 2(140)(95)\cos 78^\circ$

$L = \mathbf{152}$ m

Rounding: every side is given to one decimal place and every angle to one decimal place. Keep full accuracy in the calculator until the final step, or the last digit will drift.

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