

Law of Sines Worksheet

WORKSHEET · PAGE 1 OF 4

Missing sides and angles · answers to one decimal place

Name _____ Date _____ Score _____ / 26

THE LAW OF SINES

In any triangle, each side is proportional to the sine of the angle opposite it:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use it when you know **two angles and any side** (AAS or ASA), or **two sides and an angle opposite one of them** (SSA). If you know three sides, or two sides and the angle *between* them, the law of sines cannot start — use the law of cosines.

SECTION A Find a missing side

Use $\frac{a}{\sin A} = \frac{b}{\sin B}$. Give each answer to one decimal place.

1 $\angle A = 40^\circ$, $\angle B = 65^\circ$, $a = 12$. Find b .

Answer _____

2 $\angle A = 55^\circ$, $\angle B = 38^\circ$, $a = 20$. Find b .

Answer _____

3 $\angle A = 28^\circ$, $\angle B = 95^\circ$, $a = 9$. Find b .

Answer _____

4 $\angle A = 72^\circ$, $\angle B = 43^\circ$, $a = 15$. Find b .

Answer _____

5 $\angle A = 33^\circ$, $\angle B = 110^\circ$, $a = 7$. Find b .

Answer _____

6 $\angle A = 48^\circ$, $\angle B = 60^\circ$, $a = 25$. Find b .

Answer _____

SECTION B Find a missing angle

Rearrange to $\sin B = \frac{b \sin A}{a}$, then take the inverse sine. One decimal place.

7 $a = 10$, $b = 7$, $\angle A = 50^\circ$. Find the acute $\angle B$.

Answer _____

8 $a = 14$, $b = 9$, $\angle A = 65^\circ$. Find the acute $\angle B$.

Answer _____

9 $a = 22$, $b = 15$, $\angle A = 40^\circ$. Find the acute $\angle B$.

Answer _____

10 $a = 18$, $b = 11$, $\angle A = 75^\circ$. Find the acute $\angle B$.

Answer _____

11 $a = 30$, $b = 19$, $\angle A = 58^\circ$. Find the acute $\angle B$.

Answer _____

12 $a = 25$, $b = 16$, $\angle A = 47^\circ$. Find the acute $\angle B$.

Answer _____

Law of Sines Worksheet

WORKSHEET · PAGE 2 OF 4

The ambiguous case and triangle area

SECTION C The ambiguous case (SSA)

Two sides and a non-included angle may give two triangles, one, or none. State which, and why.

13 $a = 8$, $b = 12$, $\angle A = 35^\circ$. How many triangles are possible?

Answer _____

14 $a = 15$, $b = 10$, $\angle A = 50^\circ$. How many triangles are possible?

Answer _____

15 $a = 5$, $b = 14$, $\angle A = 40^\circ$. How many triangles are possible?

Answer _____

16 $a = 11$, $b = 12$, $\angle A = 55^\circ$. How many triangles are possible?

Answer _____

SECTION D Area of a triangle

When two sides and the angle between them are known, $\text{Area} = \frac{1}{2}ab \sin C$.

17 $a = 9$, $b = 12$, included $\angle C = 40^\circ$. Find the area.

Answer _____

18 $a = 15$, $b = 8$, included $\angle C = 112^\circ$. Find the area.

Answer _____

19 $a = 6$, $b = 6$, included $\angle C = 60^\circ$. Find the area.

Answer _____

20 $a = 20$, $b = 14$, included $\angle C = 25^\circ$. Find the area.

Answer _____

Law of Sines Worksheet

WORKSHEET · PAGE 3 OF 4

Applied problems · sketch the triangle first, then set up the ratio

SECTION E Apply it

Draw a quick sketch first, label what you know, then set up the ratio.

- 21 Two observers standing 500 m apart both sight a balloon. The angle at the first observer is 62° and at the second is 48° . How far is the balloon from the first observer?

Answer _____

- 22 A triangular plot has angles of 42° and 73° , with the side between them measuring 85 m. Find the length of the side opposite the 42° angle.

Answer _____

- 23 In triangle ABC , $\angle A = 38^\circ$, $\angle B = 51^\circ$ and $a = 24$ cm. Find the length of side c .

Answer _____

- 24 A ramp forms a triangle with angles of 27° and 118° . The side opposite the 27° angle is 4.2 m. Find the side opposite 118° .

Answer _____

Law of Sines Worksheet

WORKSHEET · PAGE 4 OF 4

Reasoning · answer in your own words

SECTION F Explain your reasoning

A sentence or two each.

- 25 Why can two sides and a non-included angle describe two different triangles, while two angles and a side never can?

Answer _____

- 26 You know all three sides of a triangle but no angles. Can the law of sines start you off? What would you use instead?

Answer _____

THE MISTAKE TO AVOID

Your calculator's $\sin^{-1}$ only ever returns an angle between 0° and 90° for a positive sine. In the SSA case the obtuse partner $180^\circ - B$ may also be valid, so always check whether it leaves a positive third angle before ruling it out.

Answer Key

ANSWER KEY · PAGE 1 OF 3

Every value recomputed from the triangle at build time.

SECTION A

$$1. b = \frac{12 \sin 65^\circ}{\sin 40^\circ} = 16.9$$

$$2. b = \frac{20 \sin 38^\circ}{\sin 55^\circ} = 15$$

$$3. b = \frac{9 \sin 95^\circ}{\sin 28^\circ} = 19.1$$

$$4. b = \frac{15 \sin 43^\circ}{\sin 72^\circ} = 10.8$$

$$5. b = \frac{7 \sin 110^\circ}{\sin 33^\circ} = 12.1$$

$$6. b = \frac{25 \sin 60^\circ}{\sin 48^\circ} = 29.1$$

SECTION B

$$7. \sin B = \frac{7 \sin 50^\circ}{10}, \text{ so } \angle B = 32.4^\circ$$

$$8. \sin B = \frac{9 \sin 65^\circ}{14}, \text{ so } \angle B = 35.6^\circ$$

$$9. \sin B = \frac{15 \sin 40^\circ}{22}, \text{ so } \angle B = 26^\circ$$

$$10. \sin B = \frac{11 \sin 75^\circ}{18}, \text{ so } \angle B = 36.2^\circ$$

$$11. \sin B = \frac{19 \sin 58^\circ}{30}, \text{ so } \angle B = 32.5^\circ$$

$$12. \sin B = \frac{16 \sin 47^\circ}{25}, \text{ so } \angle B = 27.9^\circ$$

Answer Key

ANSWER KEY · PAGE 2 OF 3

Every value recomputed from the triangle at build time.

SECTION C

13. $\sin B = 0.860$ gives $B = 59.4^\circ$ or 120.6° , and both leave a positive third angle — **two triangles**.
14. $a \geq b$, so the shorter side is opposite the known angle and only the acute solution fits — **one triangle**.
15. $\sin B = \frac{14 \sin 40^\circ}{5} = 1.800 > 1$, which is impossible — **no triangle**.
16. $\sin B = 0.894$ gives $B = 63.3^\circ$ or 116.7° , and both leave a positive third angle — **two triangles**.

SECTION D

17. Area = $\frac{1}{2}(9)(12) \sin 40^\circ = 34.7$ square units
18. Area = $\frac{1}{2}(15)(8) \sin 112^\circ = 55.6$ square units
19. Area = $\frac{1}{2}(6)(6) \sin 60^\circ = 15.6$ square units
20. Area = $\frac{1}{2}(20)(14) \sin 25^\circ = 59.2$ square units

Answer Key

ANSWER KEY · PAGE 3 OF 3

Every value recomputed from the triangle at build time.

SECTION E

$$21. \frac{500 \sin 48^\circ}{\sin 70^\circ} = 395.4 \text{ m}$$

$$22. \frac{85 \sin 42^\circ}{\sin 65^\circ} = 62.8 \text{ m}$$

$$23. \frac{24 \sin 91^\circ}{\sin 38^\circ} = 39 \text{ cm}$$

$$24. \frac{4.2 \sin 118^\circ}{\sin 27^\circ} = 8.2 \text{ m}$$

SECTION F

25. Taking the inverse sine gives an acute angle, but $\sin \theta = \sin(180^\circ - \theta)$, so an obtuse angle has the same sine. If both leave a positive third angle, both triangles exist. With two angles the third is fixed by the 180° sum, so the shape is determined.

26. No — every law of sines ratio needs an angle and the side opposite it, and here no angle is known. Use the law of cosines, $c^2 = a^2 + b^2 - 2ab \cos C$, rearranged for $\cos C$, to get the first angle; then the law of sines finishes the triangle.