Completing the Square Worksheet

Perfect squares · solving quadratics · vertex form · exact answers, no calculator

Name _____ **Date** _____ **Score** _____ **/ 28**

THE METHOD for $ax^2 + bx + c = 0$

1. If $a \neq 1$, divide every term by a.
2. Move the constant to the right side: $x^2 + Bx = C$.
3. Add $\left(\frac{B}{2}\right)^2$ to *both* sides — half the x-coefficient, squared.
4. Write the left side as a square: $\left(x + \frac{B}{2}\right)^2 = C + \left(\frac{B}{2}\right)^2$.
5. Take the square root of both sides (remember $\pm$), then solve for x.

WORKED EXAMPLE — solve $x^2 + 6x + 4 = 0$

Step 1. $a = 1$, so no dividing. Move the constant: $x^2 + 6x = -4$

Step 2. Half of 6 is 3, and $3^2 = 9$. Add 9 to both sides: $x^2 + 6x + 9 = -4 + 9$

Step 3. The left side is a perfect square: $(x + 3)^2 = 5$

Step 4. Square-root both sides: $x + 3 = \pm\sqrt{5}$

Step 5. Subtract 3: $x = -3 \pm \sqrt{5}$ ($x \approx -0.76$ or $x \approx -5.24$)

SECTION A Make a perfect square

Fill in the number that makes a perfect square trinomial, then write it as a square. Half of b, squared.

1 $x^2 + 6x + ____ = (____)^2$

2 $x^2 - 10x + ____ = (____)^2$

3 $x^2 + 14x + ____ = (____)^2$

4 $x^2 - 3x + ____ = (____)^2$

5 $x^2 + 5x + ____ = (____)^2$

6 $x^2 - x + ____ = (____)^2$

THE MISTAKE TO AVOID

Halving b only works when the x^2 coefficient is 1. For $2x^2 - 6x - 1 = 0$, divide by 2 first to get $x^2 - 3x - \frac{1}{2} = 0$, then add $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ to both sides. And always add the number to **both** sides of the equation.

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Completing the Square Worksheet

Leading coefficient 1 · show the completed square on every question

SECTION B Solve by completing the square

These factor to whole-number answers — use completing the square anyway, then check by substituting.

7 $x^2 + 6x - 16 = 0$

Solutions $x =$ _____

8 $x^2 - 4x - 21 = 0$

Solutions $x =$ _____

9 $x^2 + 10x + 9 = 0$

Solutions $x =$ _____

10 $x^2 - 8x + 12 = 0$

Solutions $x =$ _____

11 $x^2 + 2x = 24$

Solutions $x =$ _____

12 $x^2 - 12x = -27$

Solutions $x =$ _____

SECTION C Irrational answers

Leave answers in exact form with simplified radicals. Rearrange to $x^2 + bx = c$ first where needed.

13 $x^2 + 4x - 1 = 0$

Solutions $x =$ _____

14 $x^2 - 6x + 2 = 0$

Solutions $x =$ _____

15 $x^2 + 8x = 5$

Solutions $x =$ _____

16 $x^2 = 10x - 7$

Solutions $x =$ _____

17 $x^2 + 3x - 1 = 0$

Solutions $x =$ _____

18 $x^2 - 5x + 3 = 0$

Solutions $x =$ _____

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Completing the Square Worksheet

Leading coefficient not 1 and complex roots · divide by *a* before you complete the square

SECTION D Harder equations

Divide by *a* first. If the right side is negative after completing the square, the roots are complex: $\sqrt{-n} = i\sqrt{n}$.

19 $2x^2 + 8x - 10 = 0$

Solutions $x =$ _____

20 $3x^2 - 12x + 6 = 0$

Solutions $x =$ _____

21 $2x^2 - 6x - 1 = 0$

Solutions $x =$ _____

22 $4x^2 + 4x - 7 = 0$

Solutions $x =$ _____

23 $x^2 + 4x + 13 = 0$

Solutions $x =$ _____

24 $2x^2 - 4x + 6 = 0$

Solutions $x =$ _____

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Completing the Square Worksheet

Vertex form · the same method, used to graph a parabola

VERTEX FORM

Completing the square turns $y = ax^2 + bx + c$ into $y = a(x - h)^2 + k$. The vertex is $(h,\ k)$.

Example: $y = x^2 - 4x + 1 = (x^2 - 4x + 4) - 4 + 1 = (x - 2)^2 - 3$, so the vertex is $(2,\ -3)$.

When $a \neq 1$, factor a out of the x terms first: $y = 3x^2 + 6x + 1 = 3(x^2 + 2x) + 1 = 3(x + 1)^2 - 2$.

SECTION E Write in vertex form

Complete the square. Write the equation as $y = a(x - h)^2 + k$ and state the vertex.

25 $y = x^2 + 6x + 5$

Vertex form $y =$ _____

Vertex $($ _____ , _____ $)$

26 $y = x^2 - 8x + 10$

Vertex form $y =$ _____

Vertex $($ _____ , _____ $)$

27 $y = 2x^2 + 12x + 7$

Vertex form $y =$ _____

Vertex $($ _____ , _____ $)$

28 $y = x^2 + 5x + 2$

Vertex form $y =$ _____

Vertex $($ _____ , _____ $)$

Check your roots in the free Quadratic Equation Solver on notesofmath.com — choose the completing the square method to see every step.

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Answer Key

Every root checked by substituting it back into the original equation.

SECTION A Perfect squares

Add $\left(\frac{b}{2}\right)^2$.

1. $x^2 + 6x + 9 = (x+3)^2$

2. $x^2 - 10x + 25 = (x-5)^2$

3. $x^2 + 14x + 49 = (x+7)^2$

4. $x^2 - 3x + \frac{9}{4} = (x - \frac{3}{2})^2$

5. $x^2 + 5x + \frac{25}{4} = (x + \frac{5}{2})^2$

6. $x^2 - x + \frac{1}{4} = (x - \frac{1}{2})^2$

SECTION B Whole-number roots

7. $x^2 + 6x - 16 = 0$
$x^2 + 6x = 16$
add 9 : $(x+3)^2 = 25$
$x = -8$ or $x = 2$

8. $x^2 - 4x - 21 = 0$
$x^2 - 4x = 21$
add 4 : $(x-2)^2 = 25$
$x = -3$ or $x = 7$

9. $x^2 + 10x + 9 = 0$
$x^2 + 10x = -9$
add 25 : $(x+5)^2 = 16$
$x = -9$ or $x = -1$

10. $x^2 - 8x + 12 = 0$
$x^2 - 8x = -12$
add 16 : $(x-4)^2 = 4$
$x = 2$ or $x = 6$

11. $x^2 + 2x = 24$
$x^2 + 2x - 24 = 0$
$x^2 + 2x = 24$
add 1 : $(x+1)^2 = 25$
$x = -6$ or $x = 4$

12. $x^2 - 12x = -27$
$x^2 - 12x + 27 = 0$
$x^2 - 12x = -27$
add 36 : $(x-6)^2 = 9$
$x = 3$ or $x = 9$

SECTION C Irrational roots

13. $x^2 + 4x - 1 = 0$
$x^2 + 4x = 1$
add 4 : $(x+2)^2 = 5$
$x = -2 \pm \sqrt{5}$

14. $x^2 - 6x + 2 = 0$
$x^2 - 6x = -2$
add 9 : $(x-3)^2 = 7$
$x = 3 \pm \sqrt{7}$

15. $x^2 + 8x = 5$
$x^2 + 8x - 5 = 0$
$x^2 + 8x = 5$
add 16 : $(x+4)^2 = 21$
$x = -4 \pm \sqrt{21}$

16. $x^2 = 10x - 7$
$x^2 - 10x + 7 = 0$
$x^2 - 10x = -7$
add 25 : $(x-5)^2 = 18$
$x = 5 \pm 3\sqrt{2}$

17. $x^2 + 3x - 1 = 0$
$x^2 + 3x = 1$
add $\frac{9}{4}$: $(x + \frac{3}{2})^2 = \frac{13}{4}$
$x = \dfrac{-3 \pm \sqrt{13}}{2}$

18. $x^2 - 5x + 3 = 0$
$x^2 - 5x = -3$
add $\frac{25}{4}$: $(x - \frac{5}{2})^2 = \frac{13}{4}$
$x = \dfrac{5 \pm \sqrt{13}}{2}$

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Answer Key

Harder equations and vertex form

SECTION D Leading coefficient not 1, complex roots

19. $2x^2 + 8x - 10 = 0$

$\div 2$: $x^2 + 4x - 5 = 0$

$x^2 + 4x = 5$

add 4 : $(x + 2)^2 = 9$

$x = -5$ or $x = 1$

20. $3x^2 - 12x + 6 = 0$

$\div 3$: $x^2 - 4x + 2 = 0$

$x^2 - 4x = -2$

add 4 : $(x - 2)^2 = 2$

$x = 2 \pm \sqrt{2}$

21. $2x^2 - 6x - 1 = 0$

$\div 2$: $x^2 - 3x - \frac{1}{2} = 0$

$x^2 - 3x = \frac{1}{2}$

add $\frac{9}{4}$: $(x - \frac{3}{2})^2 = \frac{11}{4}$

$x = \dfrac{3 \pm \sqrt{11}}{2}$

22. $4x^2 + 4x - 7 = 0$

$\div 4$: $x^2 + x - \frac{7}{4} = 0$

$x^2 + x = \frac{7}{4}$

add $\frac{1}{4}$: $(x + \frac{1}{2})^2 = 2$

$x = \dfrac{-1 \pm 2\sqrt{2}}{2}$

23. $x^2 + 4x + 13 = 0$

$x^2 + 4x = -13$

add 4 : $(x + 2)^2 = -9$

$x = -2 \pm 3i$

24. $2x^2 - 4x + 6 = 0$

$\div 2$: $x^2 - 2x + 3 = 0$

$x^2 - 2x = -3$

add 1 : $(x - 1)^2 = -2$

$x = 1 \pm i\sqrt{2}$

SECTION E Vertex form

25. $y = x^2 + 6x + 5$

$y = (x + 3)^2 - 4$ vertex $(-3, -4)$

26. $y = x^2 - 8x + 10$

$y = (x - 4)^2 - 6$ vertex $(4, -6)$

27. $y = 2x^2 + 12x + 7$

$y = 2(x + 3)^2 - 11$ vertex $(-3, -11)$

28. $y = x^2 + 5x + 2$

$y = (x + \frac{5}{2})^2 - \frac{17}{4}$ vertex $(-\frac{5}{2}, -\frac{17}{4})$

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