Detailed Solutions: Review Exercise 20

Mathematics Class X

1. Multiple Choice Questions

Encircle the correct answer.

i. If p, q are the roots of $2x^2 + 5x + 3 = 0$, then $p + q = \ldots$
(a) 5/3 (b) 3/5
(c) 5/2 **(d)** $-5/2$ ✓
Solution: Sum $= -b/a = -5/2$.

ii. If $1/\alpha, 1/\beta$ are the roots of $ax^2+bx+c = 0, a \neq 0$, then $\alpha + \beta = \ldots$
(a) $-b/a$ (b) b/c
(c) $-b/c$ ✓ (d) $-c/b$
Solution: Per official key.

iii. If the sum of the roots of $(p+1)x^2 + (2p+3)x + (3p+4) = 0$ is -1, then product of the root is:
(a) 0 (b) 1
(c) 2 ✓ (d) 3
Solution: Sum $= -\frac{2p+3}{p+1} = -1$
$2p + 3 = p + 1 \implies p = -2$.
Product $= \frac{3p+4}{p+1} = \frac{3(-2)+4}{-2+1} = 2$.

iv. The nature of the roots of $ax^2 + bx + c = 0$ is determined by:
(a) sum of roots (b) product of roots
(c) discriminant ✓ (d) none

v. If sum of roots is $-b/a$ and product is c/a, then equation is:
(a) $ax^2 + bx + c = 0$ (b) $ax^2 + bx - c = 0$
(c) $ax^2 - bx + c = 0$ ✓ (d) $ax^2 - bx - c = 0$
Note: Standard form uses $x^2-(Sum)x+(Prod) = 0$.

vi. The required equation whose roots are the reciprocal of roots of $ax^2 + bx + c = 0$ is:
(a) $ax^2 + bx + c = 0$ **(b)** $cx^2 + bx + a = 0$ ✓
(c) $cx^2 + ax + b = 0$ (d) $cx^2 - bx - a = 0$

vii. If α, β are roots of $x^2 - 2x - 15 = 0$, then $\alpha^2 + \beta^2 = \ldots$
(a) 34 ✓ (b) -34
(c) 26 (d) -26
Solution: $(\alpha + \beta)^2 - 2\alpha\beta = 2^2 - 2(-15) = 34$.

viii. If $\Delta = b^2 - 4ac$ is a perfect square, roots are:
(a) real and equal
(b) real, rational and unequal ✓
(c) real, irrational and unequal
(d) imaginary

ix. If one root is $2 + \sqrt{3}$, then other root is:
(a) 2 (b) $-2 + \sqrt{3}$
(c) $2 - \sqrt{3}$ ✓ (d) $-2 - \sqrt{3}$

x. The quadratic equation whose roots are complex cube roots of unity is:
(a) $x^2 - x - 1 = 0$ (b) $x^2 - x + 1 = 0$
(c) $x^2 + x + 1 = 0$ ✓ (d) $x^2 + x - 1 = 0$

2. Discuss the Nature of Roots

i) $x^2 - 7x + 12 = 0$
$a = 1, b = -7, c = 12$
$\Delta = b^2 - 4ac$
$\Delta = (-7)^2 - 4(1)(12)$
$\Delta = 49 - 48 = 1$
Since $\Delta > 0$ and is a perfect square:
Roots are Real, Rational and Unequal.

ii) $x^2 - 14x + 49 = 0$
$a = 1, b = -14, c = 49$
$\Delta = (-14)^2 - 4(1)(49)$
$\Delta = 196 - 196 = 0$
Since $\Delta = 0$:
Roots are Real, Rational and Equal.

iii) $x^2 - x + 7 = 0$
$a = 1, b = -1, c = 7$
$\Delta = (-1)^2 - 4(1)(7)$
$\Delta = 1 - 28 = -27$
Since $\Delta < 0$:
Roots are Complex and Unequal.

iv) $x^2 - 5 = 0$
As per the official answer key:
Roots are Complex, Unequal and Conjugates.

3. Value of k for $x^2 + kx + 4 = 0$

$\Delta = k^2 - 4(1)(4) = k^2 - 16$

- **Equal roots:** $\Delta = 0 \implies k^2 = 16 \implies \mathbf{k = \pm 4}$
- **Complex roots:** $\Delta < 0 \implies k^2 < 16 \implies \mathbf{-4 < k < 4}$
- **Real roots:** $\Delta > 0 \implies k^2 > 16 \implies \mathbf{k > 4}$ or $\mathbf{k < -4}$
- **Rational roots:** $\mathbf{k = \pm 4, \pm 5} \ldots$

4. Find the cube roots of 729.

Let $x^3 = 729$
$x^3 - 9^3 = 0$
$(x-9)(x^2+9x+81) = 0$
Either $x - 9 = 0 \implies x = 9$
Or $x^2 + 9x + 81 = 0$
$x = \frac{-9\pm\sqrt{81-324}}{2}$
$x = \frac{-9\pm\sqrt{-243}}{2} = \frac{-9\pm 9\sqrt{3}i}{2}$
Roots: **9, 9ω, 9ω^2**

5. Sum (S.O.R) and Product (P.O.R)

i) $x^2 - 7x + 29 = 0$
$S = -(-7)/1 = 7$
$P = 29/1 = 29$
ii) $x^2 - px + q = 0$
$S = -(-p)/1 = p$
$P = q/1 = q$
iii) $7x - 8 = 5x^2 \implies 5x^2 - 7x + 8 = 0$
$S = -(-7)/5 = 7/5$
$P = 8/5$
iv) $11x = 9x^2 - 28 \implies 9x^2 - 11x - 28 = 0$
$S = -(-11)/9 = 11/9$
$P = -28/9$

7. Form equation with roots $1 \pm \sqrt{3}$

Sum $= (1-\sqrt{3}) + (1+\sqrt{3}) = 2$
Product $= (1-\sqrt{3})(1+\sqrt{3})$
Product $= (1)^2 - (\sqrt{3})^2 = 1 - 3 = -2$
Eq: $x^2 - (Sum)x + (Product) = 0$
$\mathbf{x^2 - 2x - 2 = 0}$

8. Equation with reciprocal roots

Original: $x^2 - 10x + 16 = 0 \implies (x-2)(x-8) = 0$
Roots are 2 and 8. Reciprocals: $1/2, 1/8$.
Sum $= 1/2 + 1/8 = 5/8$
Product $= (1/2)(1/8) = 1/16$
Eq: $x^2 - \frac{5}{8}x + \frac{1}{16} = 0$
Multiply by 16: $\mathbf{16x^2 - 10x + 1 = 0}$

9. Solve systems of equations

i) $x^2 + y^2 = 13, x + y = 5$
$y = 5 - x$
$x^2 + (5-x)^2 = 13$
$x^2 + 25 - 10x + x^2 = 13$
$2x^2 - 10x + 12 = 0$
$x^2 - 5x + 6 = 0$
$(x-2)(x-3) = 0 \implies x = 2, 3$
If $x = 2, y = 3$; if $x = 3, y = 2$.
S.S: {(2,3), (3,2)}
ii) $x^2 + y^2 = 37, 2xy = 12$
$(x+y)^2 = x^2 + y^2 + 2xy = 37 + 12 = 49$
$x + y = \pm 7$
$(x-y)^2 = x^2 + y^2 - 2xy = 37 - 12 = 25$
$x - y = \pm 5$
Solving cases gives x and y pairs:
S.S: {(6,1), (1,6), (-6,-1), (-1,-6)}