

Complete Solutions: Exercise 20.7

Prepared by www.notesofmath.com

Linear & Quadratic Systems

Q1: $2x - y = 3; x^2 + y^2 = 2$

1. From 1st eq:

$$y = 2x - 3$$

2. Substitute:

$$x^2 + (2x - 3)^2 = 2$$

3. Expand:

$$x^2 + 4x^2 - 12x + 9 = 2$$

$$5x^2 - 12x + 7 = 0$$

4. Factor:

$$(x - 1)(5x - 7) = 0$$

$$x = 1, \frac{7}{5}$$

5. Values:

notesofmath.com $x = 1 \Rightarrow y = -1$

$$x = \frac{7}{5} \Rightarrow y = -\frac{1}{5}$$

S.S: $\{(1, -1), (7/5, -1/5)\}$

Q2: $2x + y = 4; x^2 - 2x + y^2 = 0$

1. From 1st eq:

$$y = 4 - 2x$$

2. Substitute:

$$x^2 - 2x + (4 - 2x)^2 = 0$$

3. Expand:

$$x^2 - 2x + 16 - 16x + 4x^2 = 0$$

$$5x^2 - 18x + 16 = 0$$

4. Factor:

$$(x - 2)(5x - 8) = 0$$

$$x = 2, \frac{8}{5}$$

5. Values:

$$x = 2 \Rightarrow y = 0$$

$$x = \frac{8}{5} \Rightarrow y = \frac{4}{5}$$

S.S: $\{(2, 0), (8/5, 4/5)\}$

Q3: $4/x + 3/y = 2; 4x + 3y = 25$

1. From 2nd eq:

$$y = \frac{25 - 4x}{3}$$

2. From 1st eq:

$$4y + 3x = 2xy$$

3. Substitute:

$$4\left(\frac{25 - 4x}{3}\right) + 3x = 2x\left(\frac{25 - 4x}{3}\right)$$

4. Simplify:

$$100 - 16x + 9x = 50x - 8x^2$$

$$8x^2 - 57x + 100 = 0$$

5. Factor:

$$(x - 4)(8x - 25) = 0$$

$$x = 4, \frac{25}{8}$$

6. Values:

$$x = 4 \Rightarrow y = 3$$

$$x = \frac{25}{8} \Rightarrow y = -\frac{25}{6}$$

S.S: $\{(4, 3), (25/8, -25/6)\}$

Q4: $(x - 1)^2 + (y + 3)^2 = 25; x^2 + (y + 1)^2 = 10$

1. Expand:

$$x^2 + y^2 - 2x + 6y = 15$$

$$x^2 + y^2 + 2y = 9$$

2. Subtract:

$$-2x + 4y = 6 \Rightarrow x = 2y - 3$$

3. Substitute:

$$(2y - 3)^2 + y^2 + 2y = 9$$

4. Solve:

$$5y^2 - 10y = 0$$

$$5y(y - 2) = 0 \Rightarrow y = 0, 2$$

5. Values:

$$y = 0 \Rightarrow x = -3$$

$$y = 2 \Rightarrow x = 1$$

S.S: $\{(-3, 0), (1, 2)\}$

$$\text{Q5: } x^2 + y^2 = 25; (4x - 3y)(x - y - 5) = 0$$

Case A:

$$y = \frac{4x}{3}$$

$$x^2 + \frac{16x^2}{9} = 25$$

$$25x^2 = 225 \Rightarrow x = \pm 3, y = \pm 4$$

Case B:

$$y = x - 5$$

$$x^2 + (x - 5)^2 = 25$$

$$2x^2 - 10x = 0 \Rightarrow x = 0, 5$$

Values:

$$(0, -5), (5, 0)$$

$$\text{S.S: } \{(\pm 3, \pm 4), (0, -5), (5, 0)\}$$

$$\text{Q6: } x^2 + y^2 = 16; 2x^2 - 3xy + y^2 = 0$$

$$(2x - y)(x - y) = 0$$

Case A:

$$y = 2x \Rightarrow 5x^2 = 16 \Rightarrow x = \pm \frac{4}{\sqrt{5}}$$

Case B:

$$\text{notesofmath.com } y = x \Rightarrow 2x^2 = 16 \Rightarrow x = \pm 2\sqrt{2}$$

$$\text{S.S: } \{(\pm 2\sqrt{2}, \pm 2\sqrt{2}), (\frac{\pm 4}{\sqrt{5}}, \frac{\pm 8}{\sqrt{5}})\}$$

$$\text{Q7: } x^2 + y^2 = 5; xy = 2$$

$$y = \frac{2}{x} \Rightarrow x^2 + \frac{4}{x^2} = 5$$

$$x^4 - 5x^2 + 4 = 0$$

$$(x^2 - 4)(x^2 - 1) = 0 \Rightarrow x = \pm 2, \pm 1$$

$$\text{S.S: } \{(1, 2), (2, 1), (-1, -2), (-2, -1)\}$$

$$\text{Q8: } x^2 + xy = 5; x^2 - 2xy = 2$$

$$2x^2 + 2xy = 10$$

$$x^2 - 2xy = 2$$

$$3x^2 = 12 \Rightarrow x = \pm 2$$

$$x = 2 \Rightarrow y = \frac{1}{2}$$

$$x = -2 \Rightarrow y = -\frac{1}{2}$$

$$\text{S.S: } \{(2, 1/2), (-2, -1/2)\}$$

$$\text{Q9: } x + 4/y = 1; y + 4/x = 25$$

$$x = \frac{y - 4}{y}$$

$$y + \frac{4y}{y - 4} = 25$$

$$y^2 - 25y + 100 = 0 \Rightarrow y = 20, 5$$

$$y = 20 \Rightarrow x = \frac{4}{5}$$

$$y = 5 \Rightarrow x = \frac{1}{5}$$

$$\text{S.S: } \{(4/5, 20), (1/5, 5)\}$$

Word Problems

Q10

$$x + y = 12, \quad (x - y)^2 = 4 \Rightarrow x - y = \pm 2$$

$$x = 7, y = 5$$

Parts are 7 and 5.

Q11

$$x(x + 5) = 36 \Rightarrow x^2 + 5x - 36 = 0$$

$$(x + 9)(x - 4) = 0 \Rightarrow x = 4$$

Length = 9m, Width = 4m.

Q12

$$(y + 2)^2 + y^2 = 100$$

$$y^2 + 2y - 48 = 0 \Rightarrow y = 6$$

$$x = 8$$

Numbers are 6 and 8.

Q13

$$x^2 + (x + 3)^2 = 225$$

$$x^2 + 3x - 108 = 0 \Rightarrow x = 9$$

Base = 12cm, Perpendicular = 9cm.

Q14

$$y = 18 - x$$

$$x^2 + 144 = (18 - x)^2 \Rightarrow x = 5$$

Sides: 13cm, 13cm, 10cm.

Q15

$$x - y = 5, \quad x + y = 55$$

$$x = 30, y = 25$$

Numbers are 30 and 25.