Exercise 20.6: Step-by-Step Solutions

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Cubic Equations (Q1)

To find the remaining roots of a cubic equation ($ax^3 + bx^2 + cx + d = 0$) when one root is given, we use **Synthetic Division** to reduce it to a quadratic equation.

(i) $2x^3 - x^2 - 2x + 1 = 0$, root $x = 1$

Step 1: Synthetic Division Write coefficients: 2, -1, -2, 1. Use the root 1:

1	2	−1	−2	1
	↓	2	1	−1
	2	1	−1	|0

The remainder is 0, confirming $x = 1$ is a root. **Step 2: Depressed Equation** The coefficients (2, 1, -1) give: $2x^2 + x - 1 = 0$. **Step 3: Factorization** $2x^2 + 2x - x - 1 = 0$
$2x(x+1) - 1(x+1) = 0 \implies (x+1)(2x-1) = 0$.
Remaining roots: $-1, 1/2$

(ii) $x^3 - 4x^2 + x + 6 = 0$, root $x = 3$

Step 1: Synthetic Division

3	1	−4	1	6
	↓	3	−3	−6
	1	−1	−2	|0

Step 2: Depressed Equation $x^2 - x - 2 = 0$. **Step 3: Factorization** $x^2 - 2x + x - 2 = 0$
$x(x-2) + 1(x-2) = 0 \implies (x-2)(x+1) = 0$.
Remaining roots: $2, -1$

(iii) $x^3 - 28x + 48 = 0$, root $x = 2$

Note: Since there is no x^2 term, its coefficient is 0.
Step 1: Synthetic Division

2	1	0	−28	48
	↓	2	4	−48
	1	2	−24	|0

Step 2: Depressed Equation $x^2 + 2x - 24 = 0$. **Step 3: Factorization** Find two numbers that multiply to -24 and add to 2 (6 and -4):
$(x+6)(x-4) = 0$.
Remaining roots: $-6, 4$

Biquadratic Equations (Q2)

For x^4 equations, we perform synthetic division twice to get a quadratic.

(i) $12x^4 - 8x^3 - 7x^2 + 2x + 1 = 0$, roots $1, -1/2$

Step 1: First Division (by 1)

1	12	−8	−7	2	1
	↓	12	4	−3	−1
	12	4	−3	−1	|0

Step 2: Second Division (by -1/2)

−1/2	12	4	−3	−1
	↓	−6	1	1
	12	−2	−2	|0

Step 3: Solve Quadratic $12x^2 - 2x - 2 = 0 \implies 6x^2 - x - 1 = 0$.
$6x^2 - 3x + 2x - 1 = 0 \implies (3x+1)(2x-1) = 0$.
Remaining roots: $-1/3, 1/2$

(ii) $x^4 + 4x^2 - 5 = 0$, roots $1, -1$

Step 1: First Division (by 1)

1	1	0	4	0	−5
	↓	1	1	5	5
	1	1	5	5	|0

Step 2: Second Division (by -1)

−1	1	1	5	5
	↓	−1	0	−5
	1	0	5	|0

Step 3: Solve Quadratic $x^2 + 5 = 0 \implies x^2 = -5 \implies x = \pm\sqrt{-5}$.
Remaining roots: $\pm i\sqrt{5}$

Finding Unknowns (Q3-5)

Question 3

$2x^3 - 3mx^2 + 9 = 0$, given $x = 3$. **Step 1: Find m** Since 3 is a root, $P(3) = 0$:
$2(3)^3 - 3m(3)^2 + 9 = 0$
$54 - 27m + 9 = 0 \implies 27m = 63 \implies m = 7/3$. **Step 2: Synthetic Division** With $m = 7/3$, $-3m = -7$.

3	2	−7	0	9
	↓	6	−3	−9
	2	−1	−3	|0

Step 3: Solve $2x^2 - x - 3 = 0$
$(2x-3)(x+1) = 0 \implies x = 3/2, -1$.

Question 4

$x^3 - 3ax^2 - x + 6 = 0$, root is 1. **Step 1: Find a**
$(1)^3 - 3a(1)^2 - 1 + 6 = 0 \implies 1 - 3a - 1 + 6 = 0$.
$6 - 3a = 0 \implies a = 2$. **Step 2: Synthetic Division**
With $a = 2$, $-3a = -6$.

1	1	−6	−1	6
	↓	1	−5	−6
	1	−5	−6	|0

Step 3: Solve $x^2 - 5x - 6 = 0$
$(x-6)(x+1) = 0 \implies x = 6, -1$.

Question 5

$x^4 - ax^2 + bx + 252 = 0$, roots 6, -2. **Step 1: Simultaneous Equations**

For $x = 6$: $1296 - 36a + 6b + 252 = 0 \implies 6a - b = 258$.
For $x = -2$: $16 - 4a - 2b + 252 = 0 \implies 2a + b = 134$.
Adding: $8a = 392 \implies a = 49$.
Sub a: $2(49) + b = 134 \implies b = 36$. **Step 2: Double Synthetic Division**

6	1	0	−49	36	252
	↓	6	36	−78	−252
−2	1	6	−13	−42	|0
	↓	−2	−8	42	
	1	4	−21	|0	

Step 3: Solve $x^2 + 4x - 21 = 0$
$(x+7)(x-3) = 0 \implies x = -7, 3$.