Solutions of Exercise 20.4 Class 10 Math Sindh Board

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Question 1

If α, β are the roots of a quadratic equation. Express the following symmetric functions in terms of $(\alpha+\beta)$ and $\alpha\beta$.

(i) $(\alpha-\beta)^2$

Recall that $(\alpha-\beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$. Adding and subtracting $2\alpha\beta$:

$$\begin{aligned} &= \alpha^2 + 2\alpha\beta + \beta^2 - 4\alpha\beta \\ &= (\alpha+\beta)^2 - 4\alpha\beta \end{aligned}$$

(ii) $(\alpha^3+\beta^3)$

Using the identity $a^3+b^3 = (a+b)(a^2-ab+b^2)$:

$$\begin{aligned} &= (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2) \\ &= (\alpha+\beta)[(\alpha^2+2\alpha\beta+\beta^2)-3\alpha\beta] \\ &= (\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta] \\ &= (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) \end{aligned}$$

(iii) $\alpha^2\beta^{-1} + \beta^2\alpha^{-1}$

Rewriting with positive exponents:

$$\begin{aligned} &= \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} \\ &= \frac{\alpha^3+\beta^3}{\alpha\beta} \\ &= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta} \end{aligned}$$

(iv) $\alpha^3\beta + \alpha\beta^3$

Factor out the common term $\alpha\beta$:

$$\begin{aligned} &= \alpha\beta(\alpha^2+\beta^2) \\ &= \alpha\beta[(\alpha+\beta)^2 - 2\alpha\beta] \end{aligned}$$

(v) $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$

Taking the LCM:

$$\begin{aligned} &= \frac{\beta^3+\alpha^3}{\alpha^3\beta^3} \\ &= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3} \end{aligned}$$

(vi) $\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2$

Simplify inside the bracket:

$$\begin{aligned} &= \left(\frac{\beta-\alpha}{\alpha\beta}\right)^2 \\ &= \frac{(\alpha-\beta)^2}{(\alpha\beta)^2} \\ &= \frac{(\alpha+\beta)^2 - 4\alpha\beta}{(\alpha\beta)^2} \end{aligned}$$

Question 2

If α, β are the roots of the equation $2x^2 - 3x + 7 = 0$, find the value of following symmetric functions.
From the given equation:
$a = 2, b = -3, c = 7$.
Sum of roots:
$\alpha+\beta = -\frac{b}{a} = \frac{3}{2}$
and Product of roots: $\alpha\beta = \frac{c}{a} = \frac{7}{2}$. We will use these values of Sum and Products of roots

(i) $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^2$

$$\begin{aligned} &= \left(\frac{\alpha+\beta}{\alpha\beta}\right)^2 = \left(\frac{3/2}{7/2}\right)^2 \\ &= \left(\frac{3}{7}\right)^2 = \frac{9}{49} \end{aligned}$$

(ii) $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$

$$\begin{aligned} &= \frac{\alpha^3+\beta^3}{(\alpha\beta)^2} \\ &= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^2} \\ &= \frac{(3/2)^3 - 3(7/2)(3/2)}{(7/2)^2} \\ &= \frac{\frac{27}{8} - \frac{63}{4}}{\frac{49}{4}} = \frac{\frac{27-126}{8}}{\frac{49}{4}} \\ &= \frac{-99}{8} \times \frac{4}{49} = -\frac{99}{98} \end{aligned}$$

Question 2 (iii)

Find the value of $\frac{1}{a\alpha+1}+\frac{1}{a\beta+1}$ for the equation $2x^2-3x+7=0$.

Solution: First, find the common denominator:

$$=\frac{(a\beta+1)+(a\alpha+1)}{(a\alpha+1)(a\beta+1)}$$
$$=\frac{a(\alpha+\beta)+2}{a^2(\alpha\beta)+a(\alpha+\beta)+1}$$

From the equation $2x^2-3x+7=0$, we have: Sum $(\alpha+\beta)=\frac{3}{2}$ and Product $(\alpha\beta)=\frac{7}{2}$.

Substitute these into our expression:

$$=\frac{a(\frac{3}{2})+2}{a^2(\frac{7}{2})+a(\frac{3}{2})+1}$$

Multiply numerator and denominator by 2 to clear fractions:

$$=\frac{3a+4}{7a^2+3a+2}$$

Question 3

If α,β are roots of $px^2+qx+q=0$, find $\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}$.

Solution: Sum of roots: $\alpha+\beta=-\frac{q}{p}$

Product of roots: $\alpha\beta=\frac{q}{p}$

Express the function with a common denominator:

$$\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}=\frac{\sqrt{\alpha}}{\sqrt{\beta}}+\frac{\sqrt{\beta}}{\sqrt{\alpha}}$$
$$=\frac{(\sqrt{\alpha})^2+(\sqrt{\beta})^2}{\sqrt{\alpha}\sqrt{\beta}}$$
$$=\frac{\alpha+\beta}{\sqrt{\alpha\beta}}$$

Now, substitute the values of sum and product:

$$=\frac{-q/p}{\sqrt{q/p}}$$

Since any number x divided by its square root is $\sqrt{x}$ (i.e., $\frac{x}{\sqrt{x}}=\sqrt{x}$), let $x=q/p$:

$$=-\left(\frac{q/p}{\sqrt{q/p}}\right)=-\sqrt{\frac{q}{p}}$$

Question 4

Represent symmetric equation $\alpha+\beta=8$ graphically when α and β are roots of a quadratic equation and also plot the graph.

Solution:

- $\alpha+\beta=8$ is a **linear equation**. It represents a straight line.
- $\alpha\beta=$ constant represents a **hyperbola**.