# Detailed Solutions: Exercise 20.2

Class 10 Mathematics Sindh Board
Prepared by www.notesofmath.com

## Useful Properties of $\omega$

Recall that $\omega$ is a cube root of unity, where

$$\omega^3 = 1 \quad \text{and} \quad \omega \neq 1$$

Important results:

- $1 + \omega + \omega^2 = 0$
- $\omega^4 = \omega$
- $\omega^6 = 1$

## Q1. Find all the cube roots of

**(i)** 64
Let $x^3 = 64 = 4^3$

$$x = 4(1),\ 4\omega,\ 4\omega^2$$

**Roots:** $\boxed{4,\ 4\omega,\ 4\omega^2}$

**(ii)** $-125$
Let $x^3 = -125 = (-5)^3$

$$x = -5(1),\ -5\omega,\ -5\omega^2$$

**Roots:** $\boxed{-5,\ -5\omega,\ -5\omega^2}$

**(iii)** 216
Let $x^3 = 216 = 6^3$

$$x = 6(1),\ 6\omega,\ 6\omega^2$$

**Roots:** $\boxed{6,\ 6\omega,\ 6\omega^2}$

**(iv)** $-b^3$
Let $x^3 = -b^3$

$$x = -b(1),\ -b\omega,\ -b\omega^2$$

**Roots:** $\boxed{-b,\ -b\omega,\ -b\omega^2}$

## Q2. Evaluate

**(i)** $(1 + \omega^2)^4$
Using $1 + \omega + \omega^2 = 0$,

$$1 + \omega^2 = -\omega$$

$$(1 + \omega^2)^4 = (-\omega)^4 = \omega^4 = \omega$$

**Answer:** $\boxed{\omega}$

**(ii)** $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$

Rewriting,

$$[(1 + \omega^2) - \omega][(1 + \omega) - \omega^2]$$

Using $1 + \omega^2 = -\omega$ and $1 + \omega = -\omega^2$,

$$(-\omega - \omega)(-\omega^2 - \omega^2)$$

$$(-2\omega)(-2\omega^2) = 4\omega^3 = 4$$

**Answer:** $\boxed{4}$

**(iii)** $(2 + 5\omega + 2\omega^2)^6$

$$= \{2(1 + \omega^2) + 5\omega\}^6$$

Since $1 + \omega^2 = -\omega$,

$$(-2\omega + 5\omega)^6 = (3\omega)^6$$

$$= 3^6\omega^6 = 729$$

**Answer:** $\boxed{729}$

**(iv)** $(-1 + \sqrt{-3})^4 + (-1 - \sqrt{-3})^4$
Since

$$\frac{-1 + \sqrt{-3}}{2} = \omega, \quad \frac{-1 - \sqrt{-3}}{2} = \omega^2$$

$$= (2\omega)^4 + (2\omega^2)^4$$

$$= 16(\omega^4 + \omega^8) = 16(\omega + \omega^2)$$

$$= 16(-1) = -16$$

**Answer:** $\boxed{-16}$

## Q3. Show that

**(i)**

$$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8) = (\omega+\omega^2)^4$$

Since $\omega^4 = \omega$ and $\omega^8 = \omega^2$,

$$\text{L.H.S} = (1+\omega)^2(1+\omega^2)^2$$

$$= (-\omega^2)^2(-\omega)^2 = \omega^6 = 1$$

$$\text{R.H.S} = (\omega+\omega^2)^4 = (-1)^4 = 1$$

Hence proved.

**(ii)**

$$(a+b)(a\omega+b\omega^2)(a\omega^2+b\omega) = a^3+b^3$$

$$(a\omega+b\omega^2)(a\omega^2+b\omega) = a^2+b^2+ab(\omega+\omega^2)$$

$$= a^2-ab+b^2$$

$$(a+b)(a^2-ab+b^2) = a^3+b^3$$

Hence proved.

**(iii)**

$$(a+\omega b+\omega^2 c)(a+\omega^2 b+\omega c)$$

Expanding and simplifying,

$$= a^2+b^2+c^2-ab-bc-ca$$

Hence proved.

**(iv)**

$$\left(\frac{-1+i\sqrt{3}}{2}\right)^9 + \left(\frac{-1-i\sqrt{3}}{2}\right)^9 - 2$$

$$= \omega^9 + (\omega^2)^9 - 2$$

$$= (\omega^3)^3 + (\omega^3)^6 - 2 = 1+1-2 = 0$$

Hence proved.

**(v)**

$$(1+\omega-\omega^2)^3 - (1-\omega+\omega^2)^3$$

$$= (-2\omega^2)^3 - (-2\omega)^3$$

$$= -8\omega^6 + 8\omega^3 = -8+8 = 0$$

Hence proved.