Exercise 20.3 Solutions Class 10 Sindh Board
Prepared by www.notesofmath.com

Question 1

Without solving, find the sum and product of the roots.

(i) $x^2 - x - 1 = 0$

Comparing with $ax^2 + bx + c = 0$,

$$a = 1, \quad b = -1, \quad c = -1$$

Sum of roots

$$= -\frac{b}{a} = -\frac{-1}{1} = 1$$

Product of roots

$$= \frac{c}{a} = \frac{-1}{1} = -1$$

(ii) $2x^2 - 3x - 4 = 0$

$$a = 2, \quad b = -3, \quad c = -4$$

Sum of roots

$$= -\frac{b}{a} = -\frac{-3}{2} = \frac{3}{2}$$

Product of roots

$$= \frac{c}{a} = \frac{-4}{2} = -2$$

(iii) $x^2 - \frac{3}{4}a^2 = ax$

Rewriting in standard form,

$$x^2 - ax - \frac{3}{4}a^2 = 0$$

$$a = 1, \quad b = -a, \quad c = -\frac{3}{4}a^2$$

Sum of roots

$$= -\frac{b}{a} = a$$

Product of roots

$$= \frac{c}{a} = -\frac{3}{4}a^2$$

Question 2

Find the value of m.

(i) $x^2 + (3m - 7)x + 5m = 0$

$$a = 1, \quad b = 3m - 7, \quad c = 5m$$

Sum of roots

$$= -\frac{b}{a} = 7 - 3m$$

Product of roots

$$= \frac{c}{a} = 5m$$

Given:

$$\text{Sum} = \frac{3}{2} \times \text{Product}$$

$$7 - 3m = \frac{3}{2}(5m)$$

$$14 - 6m = 15m$$

$$14 = 21m$$

$$m = \frac{2}{3}$$

(ii) $2x^2 - 3x + 4m = 0$

$$a = 2, \quad b = -3, \quad c = 4m$$

Sum of roots

$$= -\frac{b}{a} = \frac{3}{2}$$

Product of roots

$$= \frac{c}{a} = 2m$$

Given:

$$\text{Sum} = 6 \times \text{Product}$$

$$\frac{3}{2} = 6(2m)$$

$$\frac{3}{2} = 12m$$

$$m = \frac{1}{8}$$

Question 3

Find the value of p.

(i) $x^2 - px + 6 = 0$

$$a = 1, \quad b = -p, \quad c = 6$$

Sum of roots

$$= p$$

Product of roots

$$= 6$$

Sum of squares of roots

$$= (\alpha + \beta)^2 - 2\alpha\beta$$

$$= p^2 - 12$$

Given:

$$p^2 - 12 = 13$$

$$p^2 = 25$$

$$p = \pm 5$$

(ii) $x^2 - 2px + (2p - 3) = 0$

$$a = 1, \quad b = -2p, \quad c = 2p - 3$$

Sum of roots

$$= 2p$$

Product of roots

$$= 2p - 3$$

Sum of squares of roots

$$= (2p)^2 - 2(2p - 3)$$

$$= 4p^2 - 4p + 6$$

Given:

$$4p^2 - 4p + 6 = 6$$

$$4p^2 - 4p = 0$$

$$4p(p - 1) = 0$$

$$p = 0 \quad \text{or} \quad p = 1$$

Question 4

Find the value of m.

(i) $x^2 - 5x + 2m = 0$

For the equation $ax^2 + bx + c = 0$,

$$a = 1, \quad b = -5, \quad c = 2m$$

Difference of roots is given by

$$\sqrt{\frac{b^2 - 4ac}{a^2}}$$

Given that the roots differ by 1,

$$\sqrt{(-5)^2 - 4(1)(2m)} = 1$$

$$\sqrt{25 - 8m} = 1$$

$$25 - 8m = 1$$

$$8m = 24$$

$$m = 3$$

(ii) $x^2 - 8x + m + 2 = 0$

$$a = 1, \quad b = -8, \quad c = m + 2$$

Given that the roots differ by 2,

$$\sqrt{(-8)^2 - 4(1)(m + 2)} = 2$$

$$\sqrt{64 - 4m - 8} = 2$$

$$\sqrt{56 - 4m} = 2$$

$$56 - 4m = 4$$

$$4m = 52$$

$$m = 13$$

Question 5

Find the value of k.

(i) $5x^2 - 7x + k - 2 = 0$

Let the roots be α and β.

$$a = 5, \quad b = -7, \quad c = k - 2$$

Sum of roots

$$\alpha + \beta = \frac{7}{5}$$

Product of roots

$$\alpha\beta = \frac{k - 2}{5}$$

Given:

$$2\alpha + 5\beta = 1$$

Multiply $\alpha + \beta = \frac{7}{5}$ by 2,

$$2\alpha + 2\beta = \frac{14}{5}$$

Subtract from given relation,

$$(2\alpha + 5\beta) - (2\alpha + 2\beta) = 1 - \frac{14}{5}$$

$$3\beta = -\frac{9}{5}$$

$$\beta = -\frac{3}{5}$$

$$\alpha = \frac{7}{5} + \frac{3}{5} = 2$$

Product:

$$\alpha\beta = 2 \times \left(-\frac{3}{5}\right) = -\frac{6}{5}$$

$$\frac{k - 2}{5} = -\frac{6}{5}$$

$$k - 2 = -6$$

$$k = -4$$

(ii) $3x^2 - 2x + 7k + 2 = 0$

$$a = 3, \quad b = -2, \quad c = 7k + 2$$

Sum of roots

$$\alpha + \beta = \frac{2}{3}$$

Product of roots

$$\alpha\beta = \frac{7k + 2}{3}$$

Given:

$$7\alpha - 3\beta = 18$$

Multiply sum equation by 3,

$$3\alpha + 3\beta = 2$$

Add with given equation,

$$(7\alpha - 3\beta) + (3\alpha + 3\beta) = 18 + 2$$

$$10\alpha = 20$$

$$\alpha = 2$$

$$\beta = \frac{2}{3} - 2 = -\frac{4}{3}$$

Product:

$$\alpha\beta = 2 \times \left(-\frac{4}{3}\right) = -\frac{8}{3}$$

$$\frac{7k+2}{3} = -\frac{8}{3}$$

$$7k + 2 = -8$$

$$k = -\frac{10}{7}$$

Question 6

Find the value of p, given that sum and product of the roots are equal.

(i) $(2p+3)x^2 + (7p-5)x + (3p-10) = 0$

$$a = 2p+3, \quad b = 7p-5, \quad c = 3p-10$$

Sum of roots

$$= -\frac{7p-5}{2p+3}$$

Product of roots

$$= \frac{3p-10}{2p+3}$$

Given:

$$-\frac{7p-5}{2p+3} = \frac{3p-10}{2p+3}$$

$$-(7p-5) = 3p-10$$

$$-7p+5 = 3p-10$$

$$10p = 15$$

$$p = \frac{3}{2}$$

(ii) $4x^2 - (5p+3)x + 17 - 9p = 0$

$$a = 4, \quad b = -(5p+3), \quad c = 17 - 9p$$

Sum of roots

$$= \frac{5p+3}{4}$$

Product of roots

$$= \frac{17-9p}{4}$$

Given:

$$\frac{5p+3}{4} = \frac{17-9p}{4}$$

$$5p+3 = 17-9p$$

$$14p = 14$$

$$p = 1$$