Mathematics Class 10 (Sindh Board)
Exercise 20.1 Solutions

Prepared by www.notesofmath.com

Question 2: Find the value(s) of k that ensure that following quadratic equations have (a) same solution (b) different real solutions.

(Hint: For same solution $\Delta = 0$ and for different real solutions $\Delta > 0$.)

(i) $x^2 - 3x + k = 0$
$a = 1,\ b = -3,\ c = k$
$\Delta = b^2 - 4ac$
$\Delta = 9 - 4k$

(a) Same solution ($\Delta = 0$):
$9 - 4k = 0$
$k = \frac{9}{4}$

(b) Different real solutions ($\Delta > 0$):
$9 - 4k > 0$
$k < \frac{9}{4}$

(ii) $x^2 + k = 4$
Standard form: $x^2 + (k - 4) = 0$
$a = 1,\ b = 0,\ c = k - 4$
$\Delta = -4k + 16$

(a) Same solution ($\Delta = 0$):
$k = 4$

(b) Different real solutions ($\Delta > 0$):
$k < 4$

(iii) $x^2 + kx + 2 = 0$
$a = 1,\ b = k,\ c = 2$
$\Delta = k^2 - 8$

(a) Same solution ($\Delta = 0$):
$k = \pm 2\sqrt{2}$

(b) Different real solutions ($\Delta > 0$):
$|k| > 2\sqrt{2}$

(iv) $(k - 1)x^2 - 4x + 2 = 0$
$a = k - 1,\ b = -4,\ c = 2$
$\Delta = 24 - 8k$

(a) Same solution ($\Delta = 0$):
$k = 3$

(b) Different real solutions ($\Delta > 0$):
$k < 3$

(v) $x^2 + kx + 4 = 0$
$a = 1,\ b = k,\ c = 4$
$\Delta = k^2 - 16$

(a) Same solution ($\Delta = 0$):
$k = \pm 4$

(b) Different real solutions ($\Delta > 0$):
$|k| > 4$

(vi) $9x^2 + kx = -16$
Standard form: $9x^2 + kx + 16 = 0$
$a = 9,\ b = k,\ c = 16$
$\Delta = k^2 - 576$

(a) Same solution ($\Delta = 0$):
$k = \pm 24$

(b) Different real solutions ($\Delta > 0$):
$|k| > 24$

(vii) $(k - 2)x^2 = 4x + (k + 2)$
Standard form: $(k - 2)x^2 - 4x - (k + 2) = 0$
$a = k - 2,\ b = -4,\ c = -(k + 2)$
$\Delta = 4k^2$

(a) Same solution ($\Delta = 0$):
$k = 0$

(b) Different real solutions ($\Delta > 0$):
All real values of k except 0

(viii) $x^2 + 1 = kx$
Standard form: $x^2 - kx + 1 = 0$
$a = 1,\ b = -k,\ c = 1$
$\Delta = k^2 - 4$

(a) Same solution ($\Delta = 0$):
$k = \pm 2$

(b) Different real solutions ($\Delta > 0$):
$|k| > 2$

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Question 3

Determine the value of m that makes the roots equal.

(i) $(m+1)x^2 + 2(m+3)x + (2m+3) = 0$

$a = m+1,\ b = 2(m+3),\ c = 2m+3$

For equal roots, $\Delta = 0$

$$[2(m+3)]^2 - 4(m+1)(2m+3) = 0$$

$4(m^2+6m+9) - 4(2m^2+5m+3) = 0$

$(m^2+6m+9) - (2m^2+5m+3) = 0$

$-m^2+m+6 = 0$

$m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$

Values of m: $m = 3,\ -2$

(ii) $9x^2 + mx + 16 = 0$

$a = 9,\ b = m,\ c = 16$

$$m^2 - 576 = 0$$

$m^2 = 576$

Values of m: $m = \pm 24$

Question 4

Show that the roots are real.

(Roots are real if $\Delta \geq 0$)

(i) $x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0$

$a = 1,\ b = -2\left(k + \frac{1}{k}\right),\ c = 3$

$$\Delta = 4\left(k^2 + \frac{1}{k^2} - 1\right)$$

Since $k^2 + \frac{1}{k^2} \geq 2$ for $k \neq 0$:

$\Delta \geq 4$

Roots are real.

(ii) $2nx^2 + 2(l+m+n)x + (l+m) = 0$

$a = 2n,\ b = 2(l+m+n),\ c = l+m$

$$\Delta = 4[(l+m)^2 + n^2]$$

Since squares are non-negative, $\Delta \geq 0$.

Roots are real.

Question 5

Show that the roots are rational.

(Roots are rational if Δ is a perfect square)

(i) $(l-m)x^2 + (m+n-l)x - n = 0$

$a = l-m,\ b = m+n-l,\ c = -n$

Let $X = m - l$

$$\Delta = (X-n)^2 = (m-l-n)^2$$

Roots are rational.

(ii) $(a+c-b)x^2 + 2cx + (b+c-a) = 0$

$a' = a+c-b,\ b' = 2c,\ c' = b+c-a$

$$\Delta = 4(a-b)^2 = [2(a-b)]^2$$

Roots are rational.

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Question 6

For what values of p and q the roots of quadratic equation $x^2 + (2p-4)x - (3q+5) = 0$ vanish?

Solution:

For the roots of a quadratic equation $ax^2 + bx + c = 0$ to vanish (i.e., $x_1 = 0$ and $x_2 = 0$), the coefficients b and c must both be equal to zero.

Given equation: $x^2 + (2p-4)x - (3q+5) = 0$

Here, $b = (2p-4)$ and $c = -(3q+5)$.

Step 1: Solve for p

Setting $b = 0$:

$2p - 4 = 0$

$2p = 4$

$p = \frac{4}{2}$

$\mathbf{p = 2}$

Step 2: Solve for q

Setting $c = 0$:

$-(3q+5) = 0$

$3q + 5 = 0$

$3q = -5$

$\mathbf{q = -\frac{5}{3}}$

Question 7

Show that the roots of $(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$ are real and cannot be equal unless $a = b = c$.

Solution:

First, expand the given equation:

$(x^2 - ax - bx + ab) + (x^2 - bx - cx + bc) + (x^2 - cx - ax + ca) = 0$

$3x^2 - 2(a+b+c)x + (ab+bc+ca) = 0$

Here, $A = 3$, $B = -2(a+b+c)$, and $C = (ab+bc+ca)$.

Calculate Discriminant (Δ):

$\Delta = B^2 - 4AC$

$\Delta = [-2(a+b+c)]^2 - 4(3)(ab+bc+ca)$

$\Delta = 4(a^2+b^2+c^2+2ab+2bc+2ca) - 12(ab+bc+ca)$

$\Delta = 4a^2+4b^2+4c^2+8ab+8bc+8ca-12ab-12bc-12ca$

$\Delta = 4a^2+4b^2+4c^2-4ab-4bc-4ca$

$\Delta = 2[2a^2+2b^2+2c^2-2ab-2bc-2ca]$

$\Delta = 2[(a^2-2ab+b^2)+(b^2-2bc+c^2)+(c^2-2ca+a^2)]$

$\Delta = 2[(a-b)^2+(b-c)^2+(c-a)^2]$

Conclusion:

1. Since the sum of squares is always ≥ 0, $\Delta \geq 0$. Thus, **roots are real.**

2. For roots to be equal, Δ must be 0. This only happens if:

$(a-b)^2 = 0 \Rightarrow a = b$

$(b-c)^2 = 0 \Rightarrow b = c$

$(c-a)^2 = 0 \Rightarrow c = a$

Therefore, roots are equal **only if** $a = b = c$.