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Solutions of

UNIT#19

Exercise 19.2

Class 10 Math Sindh Board

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Solved Exercises, Past Papers
MCQ Tests, Imprtant Formulas

Exercise 19.2

(Class 10, Sindh Board)

Question 1

Evaluate each of the following determinants.

(i)

$$\begin{vmatrix} -5 & -3 \\ 3 & -4 \end{vmatrix}$$

Solution:
For a 2×2 determinant,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= (-5)(-4) - (-3)(3)$$

$$= 20 + 9$$

$$= \boxed{29}$$

(ii)

$$\begin{vmatrix} -4 & 3 \\ 1 & -3 \end{vmatrix}$$

$$= (-4)(-3) - (3)(1)$$

$$= 12 - 3$$

$$= \boxed{9}$$

(iii)

$$\begin{vmatrix} -1 & -5 \\ 2 & 3 \end{vmatrix}$$

$$= (-1)(3) - (-5)(2)$$

$$= -3 + 10$$

$$= \boxed{7}$$

(iv)

$$\begin{vmatrix} 2 & -5 \\ 2 & -1 \end{vmatrix}$$

$$= (2)(-1) - (-5)(2)$$

$$= -2 + 10$$

$$= \boxed{8}$$

(v)

$$\begin{vmatrix} 3 & -3 & -4 \\ 4 & 1 & -5 \\ 0 & -1 & -4 \end{vmatrix}$$

Solution:

Expanding along the first row:

$$= 3\begin{vmatrix} 1 & -5 \\ -1 & -4 \end{vmatrix} - (-3)\begin{vmatrix} 4 & -5 \\ 0 & -4 \end{vmatrix} + (-4)\begin{vmatrix} 4 & 1 \\ 0 & -1 \end{vmatrix}$$

$$= 3(-9) + 3(-16) - 4(-4)$$

$$= -27 - 48 + 16$$

$$= \boxed{-59}$$

(vi)

$$\begin{vmatrix} -3 & 4 & -5 \\ 2 & -3 & -5 \\ 1 & 3 & 5 \end{vmatrix}$$

Expanding along first row:

$$= -3\begin{vmatrix} -3 & -5 \\ 3 & 5 \end{vmatrix} - 4\begin{vmatrix} 2 & -5 \\ 1 & 5 \end{vmatrix} - 5\begin{vmatrix} 2 & -3 \\ 1 & 3 \end{vmatrix}$$

$$= -3(0) - 4(15) - 5(9)$$

$$= -60 - 45$$

$$= \boxed{-105}$$

(vii)

$$\begin{vmatrix} 0 & 5 & -4 \\ -3 & 4 & -5 \\ 1 & 0 & -5 \end{vmatrix}$$

Expanding along first row:

$$= 0 - 5\begin{vmatrix} -3 & -5 \\ 1 & -5 \end{vmatrix} - 4\begin{vmatrix} -3 & 4 \\ 1 & 0 \end{vmatrix}$$

$$= -5(15 + 5) - 4(-4)$$

$$= -100 + 16$$

$$= \boxed{-84}$$

(viii)

$$\begin{vmatrix} 4 & 3 & -5 \\ -5 & -1 & -5 \\ 2 & 4 & 4 \end{vmatrix}$$

$$= 4\begin{vmatrix} -1 & -5 \\ 4 & 4 \end{vmatrix} - 3\begin{vmatrix} -5 & -5 \\ 2 & 4 \end{vmatrix} - 5\begin{vmatrix} -5 & -1 \\ 2 & 4 \end{vmatrix}$$

$$= 4(16) - 3(-10) - 5(-18)$$

$$= 64 + 30 + 90$$

$$= \boxed{184}$$

(ix)

$$\begin{vmatrix} 1 & 0 & 4 \\ -2 & 3 & -5 \\ 3 & 5 & 3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & -5 \\ 5 & 3 \end{vmatrix} - 0 + 4\begin{vmatrix} -2 & 3 \\ 3 & 5 \end{vmatrix}$$

$$= 1(9 + 25) + 4(-10 - 9)$$

$$= 34 - 76$$

$$= \boxed{-42}$$

(x)

$$\begin{vmatrix} 0 & a & b \\ 0 & c & d \\ 0 & x & y \end{vmatrix}$$

Solution:

Since all elements of the first column are zero,

$$|A| = \boxed{0}$$

(xi)

$$\begin{vmatrix} -2 & 0 & 0 \\ -6 & x & 1 \\ -4 & 0 & -1 \end{vmatrix}$$

Expanding along first row:

$$= -2\begin{vmatrix} x & 1 \\ 0 & -1 \end{vmatrix}$$

$$= -2(-x)$$

$$= \boxed{2x}$$

(xii)

$$\begin{vmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{vmatrix}$$

Solution:

This is a diagonal matrix.

$$|A| = 2(-3)(-4)$$

$$= \boxed{24}$$

Question 2

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

Find the minors M_{12}, M_{22}, M_{21} and the cofactors A_{12}, A_{22}, A_{21}.

Solution:

Minor M_{12}: Delete 1st row and 2nd Column

$$M_{12} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= (1)(2) - (1)(1) = 2 - 1 = 1$$

Cofactor A_{12}:

$$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 \cdot 1 = -1$$

Minor M_{22}:Delete 2nd row and 2nd column

$$M_{22} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= (2)(2) - (1)(1) = 4 - 1 = 3$$

Cofactor A_{22}:

$$A_{22} = (-1)^{2+2} M_{22} = (+1) \cdot 3 = 3$$

Minor M_{21}: Delete 2nd row and 1st column

$$M_{21} = \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix}$$

$$= (1)(2) - (1)(-1) = 2 + 1 = 3$$

Cofactor A_{21}:

$$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 \cdot 3 = -3$$

Question 3

For what value of x is the matrix

$$\begin{bmatrix} 5 - x & x + 1 \\ 2 & 4 \end{bmatrix}$$

singular?

Solution

A matrix is **singular** if its determinant is zero.

$$|A| = \begin{vmatrix} 5 - x & x + 1 \\ 2 & 4 \end{vmatrix}$$

$$= (5 - x)(4) - (x + 1)(2) = 20 - 4x - 2x - 2$$

$$= 18 - 6x$$

For singular matrix,

$$18 - 6x = 0$$

$$6x = 18 \implies x = \boxed{3}$$

Question 4

Classify the following matrices as **singular or non-singular**:

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 3 & -2 & 1 \\ 3 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 2 & -1 \\ 3 & -2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

Solution

Matrix A:

$$|A| = 0 \begin{vmatrix} -2 & 1 \\ 2 & -1 \end{vmatrix} - 2 \begin{vmatrix} 3 & 1 \\ 3 & -1 \end{vmatrix} - 1 \begin{vmatrix} 3 & -2 \\ 3 & 2 \end{vmatrix}$$

$$= 0 - 2(-3 - 3) - 1(6 + 6)$$

$$12 - 12 = 0 \implies \boxed{\text{Matrix } A \text{ is singular}}$$

Matrix B:

$$|B| = 0 \begin{vmatrix} -2 & 1 \\ 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & -2 \\ 3 & 2 \end{vmatrix}$$

$$= 0 - 2(3 - 3) - 1(6 + 6)$$

$$= -12$$

$$\Rightarrow \boxed{\text{Matrix } B \text{ is non-singular}}$$

Question 5: Find the adjoint of the following

(a)

$$A = \begin{pmatrix} 1 & 6 \\ 4 & 7 \end{pmatrix}$$

For a 2×2 matrix,

$$\text{adj}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\boxed{\text{adj}(A) = \begin{pmatrix} 7 & -6 \\ -4 & 1 \end{pmatrix}}$$

(b)

$$B = \begin{pmatrix} -3 & 2 & -5 \\ -1 & 0 & -2 \\ 3 & -4 & 1 \end{pmatrix}$$

Cofactors are found by deleting the row and column of each element.

$$C_{11} = (-1)^2 \begin{vmatrix} 0 & -2 \\ -4 & 1 \end{vmatrix} = -8,$$

$$C_{12} = (-1)^3 \begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix} = -5,$$

$$C_{13} = (-1)^4 \begin{vmatrix} -1 & 0 \\ 3 & -4 \end{vmatrix} = 4$$

$$C_{21} = (-1)^3 \begin{vmatrix} 2 & -5 \\ -4 & 1 \end{vmatrix} = 18,$$

$$C_{22} = (-1)^4 \begin{vmatrix} -3 & -5 \\ 3 & 1 \end{vmatrix} = 12,$$

$$C_{23} = (-1)^5 \begin{vmatrix} -3 & 2 \\ 3 & -4 \end{vmatrix} = -6$$

$$C_{31} = (-1)^4 \begin{vmatrix} 2 & -5 \\ 0 & -2 \end{vmatrix} = -4,$$

$$C_{32} = (-1)^5 \begin{vmatrix} -3 & -5 \\ -1 & -2 \end{vmatrix} = -1,$$

$$C_{33} = (-1)^6 \begin{vmatrix} -3 & 2 \\ -1 & 0 \end{vmatrix} = 2$$

Cofactor matrix:

$$\begin{pmatrix} -8 & -5 & 4 \\ 18 & 12 & -6 \\ -4 & -1 & 2 \end{pmatrix}$$

Adjoint is the transpose:

$$\boxed{\text{adj}(B) = \begin{pmatrix} -8 & 18 & -4 \\ -5 & 12 & -1 \\ 4 & -6 & 2 \end{pmatrix}}$$

(c)

$$C = \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix}$$

$$C_{11} = (-1)^2 \begin{vmatrix} 2 & -6 \\ 0 & 6 \end{vmatrix} = 12,$$

$$C_{12} = (-1)^3 \begin{vmatrix} 6 & -6 \\ -3 & 6 \end{vmatrix} = -18,$$

$$C_{13} = (-1)^4 \begin{vmatrix} 6 & 2 \\ -3 & 0 \end{vmatrix} = 6$$

$$C_{21} = (-1)^3 \begin{vmatrix} -2 & 0 \\ 0 & 6 \end{vmatrix} = 12,$$

$$C_{22} = (-1)^4 \begin{vmatrix} 0 & 0 \\ -3 & 6 \end{vmatrix} = 0,$$

$$C_{23} = (-1)^5 \begin{vmatrix} 0 & -2 \\ -3 & 0 \end{vmatrix} = 6$$

$$C_{31} = (-1)^4 \begin{vmatrix} -2 & 0 \\ 2 & -6 \end{vmatrix} = 12,$$

$$C_{32} = (-1)^5 \begin{vmatrix} 0 & 0 \\ 6 & -6 \end{vmatrix} = 0,$$

$$C_{33} = (-1)^6 \begin{vmatrix} 0 & -2 \\ 6 & 2 \end{vmatrix} = 12$$

Cofactor matrix:

$$\begin{pmatrix} 12 & -18 & 6 \\ 12 & 0 & 6 \\ 12 & 0 & 12 \end{pmatrix}$$

Adjoint:

$$\boxed{\text{adj}(C) = \begin{pmatrix} 12 & 12 & 12 \\ -18 & 0 & 0 \\ 6 & 6 & 12 \end{pmatrix}}$$

Question 6

Verify $A(\text{adj } A) = |A|I$, where $A = \begin{pmatrix} 2 & 3 \\ -4 & -6 \end{pmatrix}$.

1. Find $|A|$:

$|A| = (2)(-6) - (3)(-4) = -12 + 12 = 0$

2. Find adj A:

$$\text{adj } A = \begin{pmatrix} -6 & -3 \\ 4 & 2 \end{pmatrix}$$

3. Verification:

$$A(\text{adj } A) = \begin{pmatrix} 2 & 3 \\ -4 & -6 \end{pmatrix} \begin{pmatrix} -6 & -3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Since $|A|I = 0(I) = \mathbf{0}$, verified.

Question 7 Find Inverse by Adjoint Method

(i) $A = \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix}$

$|A| = (3)(-1) - (2)(-2) = 1$
adj $A = \begin{pmatrix} -1 & -2 \\ 2 & 3 \end{pmatrix}$

$$\boxed{A^{-1} = \begin{pmatrix} -1 & -2 \\ 2 & 3 \end{pmatrix}}$$

(ii) $B = \begin{pmatrix} 3 & 6 \\ 5 & 10 \end{pmatrix}$

$|B| = (3)(10) - (6)(5) = 0$

$$\boxed{\text{Inverse does not exist (Singular)}}$$

(iii) $C = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$

$|C| = 2(4-1) - 1(6-2) + 1(3-4) = 1$
Cofactors:

$$\begin{aligned} C_{11} &= 3, & C_{12} &= -4, & C_{13} &= -1 \\ C_{21} &= -1, & C_{22} &= 2, & C_{23} &= 0 \\ C_{31} &= -1, & C_{32} &= 1, & C_{33} &= 1 \end{aligned}$$

$$\boxed{C^{-1} = \begin{pmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}}$$

(iv) $D = \begin{pmatrix} 1 & 0 & 1 \\ -4 & 1 & -1 \\ 6 & -2 & 1 \end{pmatrix}$

$|D| = 1(1-2) - 0 + 1(8-6) = 1$
Cofactors:

$$\begin{aligned} D_{11} &= -1, & D_{12} &= -2, & D_{13} &= 2 \\ D_{21} &= -2, & D_{22} &= -5, & D_{23} &= 2 \\ D_{31} &= -1, & D_{32} &= -3, & D_{33} &= 1 \end{aligned}$$

$$\boxed{D^{-1} = \begin{pmatrix} -1 & -2 & -1 \\ -2 & -5 & -3 \\ 2 & 2 & 1 \end{pmatrix}}$$

(v) $E = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{pmatrix}$

$|E| = 1(5) + 1(6) + 3(-2) = 5$
Cofactors:

$$\begin{aligned} E_{11} &= 5, & E_{12} &= -6, & E_{13} &= -2 \\ E_{21} &= -5, & E_{22} &= 7, & E_{23} &= 4 \\ E_{31} &= -5, & E_{32} &= 4, & E_{33} &= 3 \end{aligned}$$

$$\boxed{E^{-1} = \frac{1}{5} \begin{pmatrix} 5 & -5 & -5 \\ -6 & 7 & 4 \\ -2 & 4 & 3 \end{pmatrix}}$$

Question 8

(i) Matrix Inversion Method

Equations: $2x + 3y = 14$
and $-4x + y = 28$.
Step 1: Write in $AX = B$ form.

$$\begin{pmatrix} 2 & 3 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 28 \end{pmatrix}$$

Step 2: Find $|A|$.

$$|A| = (2 \times 1) - (3 \times -4) = 2 + 12 = 14$$

Step 3: Find adj A.

$$\text{adj } A = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}$$

Step 4: Solve $X = A^{-1}B$
$= \frac{1}{|A|}$**(adj A)B.**

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 14 \\ 28 \end{pmatrix}$$
$$= \frac{1}{14} \begin{pmatrix} (1 \times 14) + (-3 \times 28) \\ (4 \times 14) + (2 \times 28) \end{pmatrix}$$
$$= \frac{1}{14} \begin{pmatrix} 14 - 84 \\ 56 + 56 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} -70 \\ 112 \end{pmatrix}$$

$$\boxed{x = -5, y = 8}$$

(ii) Cramer's Rule

Equations: $2x - 4y = -12$
and $3x + 2y = 0$.
Step 1:
Find determinant D (Coefficients).

$$D = \begin{vmatrix} 2 & -4 \\ 3 & 2 \end{vmatrix} = (2 \times 2) - (-4 \times 3) = 16$$

Step 2: Find D_x (Replace x-col with constants).

$$D_x = \begin{vmatrix} -12 & -4 \\ 0 & 2 \end{vmatrix}$$

$$= (-12 \times 2) - (-4 \times 0) = -24$$

Step 3: Find D_y (Replace y-col with constants).

$$D_y = \begin{vmatrix} 2 & -12 \\ 3 & 0 \end{vmatrix} = (2 \times 0) - (-12 \times 3) = 36$$

Step 4: Solve for variables.

$$x = \frac{D_x}{D} = \frac{-24}{16} = -1.5,$$

$$y = \frac{D_y}{D} = \frac{36}{16} = 2.25$$

$$\boxed{x = -\frac{3}{2}, y = \frac{9}{4}}$$