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Solutions of

UNIT #19

Exercise 19.1

Class 10 Math Sindh Board

NotesOfMath.com

Solved Exercises, Past Papers
MCQ Tests, Imprtant Formulas

Exercise 19.1

Question 1

Specify the type of each of the following matrices.

(i)

$$\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$$

Solution: The given matrix has 2 rows and 2 columns.

$\Rightarrow$ It is a square matrix of order 2×2.

(ii)

$$\begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$$

Solution: The matrix has 3 rows and 1 column.

$\Rightarrow$ It is a column matrix.

(iii)

$$\begin{bmatrix} -2 & 0 \end{bmatrix}$$

Solution: The matrix has 1 row and 2 columns.

$\Rightarrow$ It is a row matrix.

(iv)

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

Solution: All non-diagonal elements are zero and diagonal elements are equal.

$\Rightarrow$ It is a scalar matrix.

(v)

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

Solution: All non-diagonal elements are zero.

$\Rightarrow$ It is a diagonal matrix.

Question 2

What is the order of each of the following matrix?

(i)

$$\begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 3 \\ -1 & -3 & 2 \end{bmatrix}$$

Solution: Number of rows $= 3$, number of columns $= 3$.

$\Rightarrow$ Order $= 3 \times 3$

(ii)

$$\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}$$

Solution: Number of rows $= 3$, number of columns $= 2$.

$\Rightarrow$ Order $= 3 \times 2$

(iii)

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

Solution: Number of rows $= 2$, number of columns $= 3$.

$\Rightarrow$ Order $= 2 \times 3$

(iv)

$$\begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$$

Solution: Number of rows $= 3$, number of columns $= 1$.

$$\Rightarrow \text{Order} = 3 \times 1$$

(v)

$$[0]$$

Solution: Number of rows $= 1$, number of columns $= 1$.

$$\Rightarrow \text{Order} = 1 \times 1$$

Question 3

Check whether the following matrices are symmetric or skew-symmetric.

(i)

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 5 \end{bmatrix}$$

Solution: Transpose of A is

$$A^T = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 5 \end{bmatrix}$$

Since $A = A^T$,

$$\Rightarrow \text{A is a symmetric matrix.}$$

(ii)

$$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

Solution: Transpose of A is

$$A^T = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$$

Since $A^T = -A$,

$$\Rightarrow \text{A is a skew-symmetric matrix.}$$

(iii)

$$\begin{bmatrix} 1 & 7 & 3 \\ 7 & 4 & -5 \\ 3 & -5 & 6 \end{bmatrix}$$

Solution: Transpose is equal to the original matrix.

$$\Rightarrow \text{It is a symmetric matrix.}$$

(iv)

$$\begin{bmatrix} 0 & -6 & 4 \\ -6 & 0 & 7 \\ 4 & 7 & 0 \end{bmatrix}$$

Solution: Transpose is equal to the original matrix.

$$\Rightarrow \text{It is a symmetric matrix.}$$

Question 4

If

$$A=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix},\quad B=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix},\quad C=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix},$$

$$D=\begin{bmatrix}1 & 2 & 3\\ 1 & 2 & 3\\ 1 & 2 & 3\end{bmatrix}\quad \text{and}\quad E=\begin{bmatrix}1 & 2 & -1\\ -1 & 0 & 2\\ 2 & 1 & 0\end{bmatrix}$$

Compute the following (if possible):

(i) $A+C$

$$A+C=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}+\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}=\begin{bmatrix}3 & 2\\ -1 & 3\end{bmatrix}$$

(ii) $A+E$

Order of $A=2\times 2$ and order of $E=3\times 3$.

$\Rightarrow$ Addition is not possible.

(iii) $B-D$

Order of $B=2\times 2$ and order of $D=3\times 3$.

$\Rightarrow$ Subtraction is not possible.

(iv) $2B+3A$

$$2B=\begin{bmatrix}4 & -2\\ 2 & 0\end{bmatrix},\quad 3A=\begin{bmatrix}3 & -3\\ 0 & 3\end{bmatrix}$$

$$2B+3A=\begin{bmatrix}7 & -5\\ 2 & 3\end{bmatrix}$$

(v) $B-C$

$$B-C=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}-\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}=\begin{bmatrix}0 & -4\\ 2 & -2\end{bmatrix}$$

(vi) A^2

$$A^2=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -2\\ 0 & 1\end{bmatrix}$$

(vii) B^2

$$B^2=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}3 & -2\\ 2 & -1\end{bmatrix}$$

(viii) $D+E$

$$D+E=\begin{bmatrix}1 & 2 & 3\\ 1 & 2 & 3\\ 1 & 2 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & -1\\ -1 & 0 & 2\\ 2 & 1 & 0\end{bmatrix}=\begin{bmatrix}2 & 4 & 2\\ 0 & 2 & 5\\ 3 & 3 & 3\end{bmatrix}$$

Exercise 19.1 — Matrices

Question 5 (Clear Step-by-Step Solution)

Given

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 2 & 3 \\ -2 & 4 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

(i) Find $A + B$

$$A + B = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 2 & 3 \\ -2 & 4 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -1+4 & 0+2 & 1+3 \\ 2+(-2) & 1+4 & 0+1 \\ 3+3 & 2+2 & -1+1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 5 & 1 \\ 6 & 4 & 0 \end{bmatrix}$$

(ii) Find $A - B$

$$A - B = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 3 \\ -2 & 4 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -1-4 & 0-2 & 1-3 \\ 2-(-2) & 1-4 & 0-1 \\ 3-3 & 2-2 & -1-1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -5 & -2 & -2 \\ 4 & -3 & -1 \\ 0 & 0 & -2 \end{bmatrix}$$

(iii) Find $3A + 2B$

$$3A = \begin{bmatrix} -3 & 0 & 3 \\ 6 & 3 & 0 \\ 9 & 6 & -3 \end{bmatrix}, \quad 2B = \begin{bmatrix} 8 & 4 & 6 \\ -4 & 8 & 2 \\ 6 & 4 & 2 \end{bmatrix}$$

$$3A + 2B = \begin{bmatrix} -3 & 0 & 3 \\ 6 & 3 & 0 \\ 9 & 6 & -3 \end{bmatrix} + \begin{bmatrix} 8 & 4 & 6 \\ -4 & 8 & 2 \\ 6 & 4 & 2 \end{bmatrix}$$

$$3A + 2B = \begin{bmatrix} -3+8 & 0+4 & 3+6 \\ 6+(-4) & 3+8 & 0+2 \\ 9+6 & 6+4 & -3+2 \end{bmatrix}$$

$$3A + 2B = \begin{bmatrix} 5 & 4 & 9 \\ 2 & 11 & 2 \\ 15 & 10 & -1 \end{bmatrix}$$

(iv) Find AB

$$AB = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 3 \\ -2 & 4 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -4+0+3 & -2+0+2 & -3+0+1 \\ 8-2+0 & 4+4+0 & 6+1+0 \\ 12-4-3 & 6+8-2 & 9+2-1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 0 & -2 \\ 6 & 8 & 7 \\ 5 & 12 & 10 \end{bmatrix}$$

(v) Find BA

$$BA = \begin{bmatrix} 4 & 2 & 3 \\ -2 & 4 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -4+4+9 & 0+2+6 & 4+0-3 \\ 2+8+3 & 0+4+2 & -2+0-1 \\ -3+4+3 & 0+2+2 & 3+0-1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 9 & 8 & 1 \\ 13 & 6 & -3 \\ 4 & 4 & 2 \end{bmatrix}$$

Question 6
Given

$$A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 0 \\ 7 & -4 \end{bmatrix}$$

(i) Verify $A + B = B + A$

$$A + B = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 7 & -4 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2-3 & -1+0 \\ -7+7 & 4-4 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$B + A = \begin{bmatrix} -3 & 0 \\ 7 & -4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$$

$$B + A = \begin{bmatrix} -3+2 & 0-1 \\ 7-7 & -4+4 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$A + B = B + A$$

Question 7
Given

$$A = \begin{bmatrix} 8 & 2 & -1 \\ 2 & 1 & 2 \\ 2 & 3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 & -4 \\ 2 & 5 & 7 \\ 0 & 4 & -3 \end{bmatrix}$$

Find AB

$$AB = \begin{bmatrix} 8 & 2 & -1 \\ 2 & 1 & 2 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 & -4 \\ 2 & 5 & 7 \\ 0 & 4 & -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8+4+0 & 32+10-4 & -32+14+3 \\ 2+2+0 & 8+5+8 & -8+7-6 \\ 2+6+0 & 8+15+0 & -8+21+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 12 & 38 & -15 \\ 4 & 21 & -7 \\ 8 & 23 & 13 \end{bmatrix}$$

Find BA

$$BA = \begin{bmatrix} 1 & 4 & -4 \\ 2 & 5 & 7 \\ 0 & 4 & -3 \end{bmatrix} \begin{bmatrix} 8 & 2 & -1 \\ 2 & 1 & 2 \\ 2 & 3 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 8+8-8 & 2+4-12 & -1+8+0 \\ 16+10+14 & 4+5+21 & -2+10+0 \\ 0+8-6 & 0+4-9 & 0+8+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 8 & -6 & 7 \\ 40 & 30 & 8 \\ 2 & -5 & 8 \end{bmatrix}$$

$$AB \neq BA$$

Question 8: Find 2A-3B+4C
Given

$$A = \begin{bmatrix} -3 & 0 \\ 7 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ -2 & -4 \end{bmatrix}$$

$$2A = \begin{bmatrix} -6 & 0 \\ 14 & -8 \end{bmatrix}, \quad -3B = \begin{bmatrix} -6 & 3 \\ 21 & -12 \end{bmatrix}, \quad 4C = \begin{bmatrix} 4 & 0 \\ -8 & -16 \end{bmatrix}$$

$$2A - 3B + 4C = \begin{bmatrix} -6-6+4 & 0+3+0 \\ 14+21-8 & -8-12-16 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 3 \\ 27 & -36 \end{bmatrix}$$

Question 9: Find the Values of a,b,c and d

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a+3 & b-1 \\ c+1 & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a + 3 = 1 \Rightarrow a = -2$$
$$b - 1 = 0 \Rightarrow b = 1$$
$$c + 1 = 0 \Rightarrow c = -1$$
$$d = 1$$

Question 10: Find the Values of a,b,c,d,x,y,z

$$\begin{bmatrix} x & -1 & y \\ 2 & 0 & 3 \\ z & 3 & 2 \end{bmatrix} = \begin{bmatrix} 3 & a & x \\ b & 0 & c \\ 2 & 3 & d \end{bmatrix}$$

$$x = 3, \quad a = -1, \quad y = x$$
$$b = 2, \quad c = 3$$
$$z = 2, \quad d = 2$$

Question 11: Evaluate Possible Products

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \; C = \begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & -4 \\ 3 & 1 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2-2 & 1+0 & 0+2 \\ 6+1 & 3+0 & 0-1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 7 & 3 & -1 \end{bmatrix}$$

$$BC = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & -4 \\ 3 & 1 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & -2 & 0 \\ -3 & -2 & 0 \end{bmatrix}}$$

Why other products are not possible:

- AC: A is 2×2, C is 3×3 $\rightarrow$ Inner dimensions don't match $(2\neq 3)$
- BA: B is 2×3, A is 2×2 $\rightarrow$ Inner dimensions don't match $(3\neq 2)$
- CB: C is 3×3, B is 2×3 $\rightarrow$ Inner dimensions don't match $(3\neq 2)$
- CA: C is 3×3, A is 2×2 $\rightarrow$ Inner dimensions don't match $(3\neq 2)$

Question 12: Determine

$$\begin{bmatrix}1 & 0 & 3\\ 2 & 1 & 2\\ 1 & 3 & 1\end{bmatrix}\begin{bmatrix}2 & 2 & 0\\ 1 & 3 & 2\\ 3 & 2 & 0\end{bmatrix}$$
$$=\begin{bmatrix}2+0+9 & 2+0+6 & 0+0+0\\ 4+1+6 & 4+3+4 & 0+2+0\\ 2+3+3 & 2+9+2 & 0+6+0\end{bmatrix}$$
$$=\begin{bmatrix}11 & 8 & 0\\ 11 & 11 & 2\\ 8 & 13 & 6\end{bmatrix}$$

Question 13. If

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 2 \end{bmatrix}$$

and

$$C = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 2 & 3 \end{bmatrix}$$

then verify that

(i) $A(B+C) = AB + AC$

Solution:

$$B + C = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 3 & 5 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 10 & 3 & 11 \\ -1 & 6 & 8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 9 & -2 & 2 \\ 4 & 3 & 4 \end{bmatrix}, \quad AC = \begin{bmatrix} 1 & 5 & 9 \\ -5 & 3 & 4 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 10 & 3 & 11 \\ -1 & 6 & 8 \end{bmatrix}$$

$$\therefore\ A(B+C) = AB + AC$$

(ii) $(B+C)A = BA + CA$

Solution:

$(B + C)$ is of order 2×3 and A is 2×2. Multiplication is not defined.

$$\therefore\ (B+C)A \text{ is not defined.}$$

Question 14. If

$$A = \begin{bmatrix} 6 & 5 & 2 \\ 4 & 6 & 3 \\ 0 & 1 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 8 & 0 \\ 5 & 1 & 2 \\ 3 & 6 & 4 \end{bmatrix}$$

and

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

(a) $A + B = B + A$

$$A + B = \begin{bmatrix} 12 & 13 & 2 \\ 9 & 7 & 5 \\ 3 & 7 & 11 \end{bmatrix} = B + A$$

$$\therefore\ A + B = B + A$$

(b) $(A+B)+C = A+(B+C)$

$$(A+B)+C = \begin{bmatrix} 13 & 15 & 5 \\ 10 & 9 & 8 \\ 4 & 9 & 14 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} 13 & 15 & 5 \\ 10 & 9 & 8 \\ 4 & 9 & 14 \end{bmatrix}$$

$$\therefore\ (A+B)+C = A+(B+C)$$

(c) $(A+B)C = AC + BC$

$$(A+B)C = \begin{bmatrix} 27 & 54 & 81 \\ 21 & 42 & 63 \\ 21 & 42 & 63 \end{bmatrix}$$

$$AC + BC = \begin{bmatrix} 27 & 54 & 81 \\ 21 & 42 & 63 \\ 21 & 42 & 63 \end{bmatrix}$$

$$\therefore\ (A+B)C = AC + BC$$

(d) $A(B+C) = AB + AC$

$$\therefore\ A(B+C) = AB + AC$$

Question 15. If

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 2 & -1 \\ 2 & 0 & 2 \end{bmatrix}$$

and

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

Show that $A(BC) = (AB)C$.

Solution:

$$BC = \begin{bmatrix} 2 & 2 & 5 \\ -2 & 2 & -3 \\ 4 & 0 & 6 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 4 & 10 & 19 \\ 8 & 0 & 14 \\ 4 & 4 & 6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 10 & 1 \\ 4 & 0 & 4 \\ 2 & 4 & 2 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 4 & 10 & 19 \\ 8 & 0 & 14 \\ 4 & 4 & 6 \end{bmatrix}$$

$$\therefore\ A(BC) = (AB)C$$