بسم الله الرحمن الرحيم

Solutions of

UNIT#19

Review Exercise 19

Class 10 Math Sindh Board

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Review Exercise 19 - Solutions

Class 10 Math Sindh Board (www.notesofmath.com)

1. Tick the correct option

i. If m denotes the number of rows and n denotes the number of columns such that $m = n$, then the matrix is called ______ matrix.

(a) Rectangular
(b) Equal
(c) Square
(d) Null

ii. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ then A^2 is ______.

(a) I_2
(b) I_3
(c) $-I_2$
(d) O

iii. If A is any square matrix such that $A^t = -A$, then A is said to be:

(a) Diagonal matrix
(b) Scalar matrix
(c) Symmetric matrix
(d) Skew Symmetric matrix

iv. If A, B and C are matrices of same order then $(ABC)^t =$

(a) $A^t \cdot B^t \cdot C^t$
(b) $C^t B^t A^t$
(c) $C^t A^t B^t$
(d) $(B^t A^t) C^t$

v. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & i^2 \\ -i^2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ then:

(a) $A^2 = -I_2$
(b) $B^2 = -I_2$
(c) $C^2 = -I_2$
(d) All of them

vi. For two matrices A and B if $A = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ and $B = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$ then $AB =$

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
(d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

vii. If $2\begin{bmatrix} x \\ 3 \\ 4 \end{bmatrix} + 3\begin{bmatrix} -1 \\ y \\ 0 \end{bmatrix} = 4\begin{bmatrix} 0 \\ 4 \\ z \end{bmatrix}$ then the values of x, y and z respectively are:

(a) $2, \frac{3}{2}, \frac{10}{3}$
(b) $\frac{3}{2}, 2, \frac{10}{3}$
(c) $\frac{3}{2}, \frac{10}{3}, 2$
(d) $1, 2, 3$

viii. For matrix A, $(A^{-1})^{-1} =$

(a) A^{-2}
(b) A
(c) A^{-1}
(d) A^2

ix. Find x if $\begin{vmatrix} 5 & 1 \\ 2 & x \end{vmatrix} = x + 4$

(a) $\frac{3}{2}$
(b) $\frac{2}{3}$
(c) 0
(d) None of them

x. If the matrix $\begin{bmatrix} \lambda & -3 & 4 \\ -3 & 0 & 1 \\ -1 & 3 & 2 \end{bmatrix}$ is invertible then $\lambda \neq$ ______.

(a) -15
(b) -17
(c) -16
(d) None of these

Question 2: Definitions

Define the row and column of a matrix.

- **Row:** The horizontal arrangement of elements in a matrix.
  Example: In $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, the elements 1, 2, and 3 form a row.
- **Column:** The vertical arrangement of elements in a matrix.
  Example: In $\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$, the elements 1, 4, and 7 form a column.

Question 3: Order of Matrices

The order of a matrix is given by $m \times n$, where m is the number of rows and n is the number of columns.

i. $[1 \quad 5 \quad 8]$
Number of rows = 1, Columns = 3
Order: 1×3

ii. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 8 \end{bmatrix}$
Number of rows = 2, Columns = 3
Order: 2×3

iii. $\begin{bmatrix} 7 & 2 \\ 9 & 3 \\ 8 & 1 \end{bmatrix}$
Number of rows = 3, Columns = 2
Order: 3×2

iv. $[a + b + c]$
Number of rows = 1, Columns = 1
Order: 1×1

Question 4: Types of Matrices

i. **Square Matrix:** A matrix in which the number of rows is equal to the number of columns ($m = n$).
Example: $A = \begin{bmatrix} 4 & 7 \\ 2 & 9 \end{bmatrix}$ (Order 2×2)

ii. **Rectangular Matrix:** A matrix in which the number of rows is not equal to the number of columns ($m \neq n$).
Example: $B = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 3 & 6 \end{bmatrix}$ (Order 2×3)

iii. **Diagonal Matrix:** A square matrix in which all the elements except at least one of the leading diagonal elements are zero.
Example: $C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

iv. **Scalar Matrix:** A diagonal matrix in which all the leading diagonal elements are equal (and non-zero).
Example: $D = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

v. **Symmetric Matrix:** A square matrix A where $A^t = A$.
Example: $E = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$. Here, $E^t = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = E$.

vi. **Skew Symmetric Matrix:** A square matrix A where $A^t = -A$. The main diagonal elements are always zero.
Example: $F = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$. Here, $F^t = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -F$.

Question 5: Matrix Operations

Given: $A = \begin{bmatrix} 1 & 8 & 9 \\ 2 & 1 & 0 \\ -2 & 1 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 & 8 \\ 1 & 3 & 4 \\ 7 & 2 & 4 \end{bmatrix}$

i. $A + B$
$$= \begin{bmatrix} 1+2 & 8+5 & 9+8 \\ 2+1 & 1+3 & 0+4 \\ -2+7 & 1+2 & 7+4 \end{bmatrix} = \begin{bmatrix} 3 & 13 & 17 \\ 3 & 4 & 4 \\ 5 & 3 & 11 \end{bmatrix}$$

ii. $A - B$
$$= \begin{bmatrix} 1-2 & 8-5 & 9-8 \\ 2-1 & 1-3 & 0-4 \\ -2-7 & 1-2 & 7-4 \end{bmatrix} = \begin{bmatrix} -1 & 3 & 1 \\ 1 & -2 & -4 \\ -9 & -1 & 3 \end{bmatrix}$$

iii. AB **(Multiplication)**
Row-by-column calculation:
$c_{11} = 1(2) + 8(1) + 9(7) = 73$
$c_{12} = 1(5) + 8(3) + 9(2) = 47$
$c_{13} = 1(8) + 8(4) + 9(4) = 76$
$c_{21} = 2(2) + 1(1) + 0(7) = 5$
$c_{22} = 2(5) + 1(3) + 0(2) = 13$
$c_{23} = 2(8) + 1(4) + 0(4) = 20$
$c_{31} = -2(2) + 1(1) + 7(7) = 46$
$c_{32} = -2(5) + 1(3) + 7(2) = 7$
$c_{33} = -2(8) + 1(4) + 7(4) = 16$
$$\mathbf{AB} = \begin{bmatrix} 73 & 47 & 76 \\ 5 & 13 & 20 \\ 46 & 7 & 16 \end{bmatrix}$$

iv. BA **(Multiplication)**
Row-by-column calculation:
$d_{11} = 2(1) + 5(2) + 8(-2) = -4$
$d_{12} = 2(8) + 5(1) + 8(1) = 29$
$d_{13} = 2(9) + 5(0) + 8(7) = 74$
$d_{21} = 1(1) + 3(2) + 4(-2) = -1$
$d_{22} = 1(8) + 3(1) + 4(1) = 15$
$d_{23} = 1(9) + 3(0) + 4(7) = 37$
$d_{31} = 7(1) + 2(2) + 4(-2) = 3$
$d_{32} = 7(8) + 2(1) + 4(1) = 62$
$d_{33} = 7(9) + 2(0) + 4(4) = 79$
$$\mathbf{BA} = \begin{bmatrix} -4 & 29 & 74 \\ -1 & 15 & 37 \\ 3 & 62 & 79 \end{bmatrix}$$

v. $5A$
$$= \begin{bmatrix} 5(1) & 5(8) & 5(9) \\ 5(2) & 5(1) & 5(0) \\ 5(-2) & 5(1) & 5(7) \end{bmatrix} = \begin{bmatrix} 5 & 40 & 45 \\ 10 & 5 & 0 \\ -10 & 5 & 35 \end{bmatrix}$$

vi. $7B$

$$= \begin{bmatrix} 7(2) & 7(5) & 7(8) \\ 7(1) & 7(3) & 7(4) \\ 7(7) & 7(2) & 7(4) \end{bmatrix} = \begin{bmatrix} 14 & 35 & 56 \\ 7 & 21 & 28 \\ 49 & 14 & 28 \end{bmatrix}$$

vii. $8A - 9B$

$$8A = \begin{bmatrix} 8 & 64 & 72 \\ 16 & 8 & 0 \\ -16 & 8 & 56 \end{bmatrix}, 9B = \begin{bmatrix} 18 & 45 & 72 \\ 9 & 27 & 36 \\ 63 & 18 & 36 \end{bmatrix}$$

$$8A - 9B = \begin{bmatrix} -10 & 19 & 0 \\ 7 & -19 & -36 \\ -79 & -10 & 20 \end{bmatrix}$$

Question 6:Evaluate Determinant

Evaluate the determinant: $\Delta = \begin{vmatrix} 1 & 8 & 9 \\ 2 & 0 & -1 \\ -7 & 8 & -10 \end{vmatrix}$

Solution: Expanding along the first row (R_1):

$$\Delta = 1\begin{vmatrix} 0 & -1 \\ 8 & -10 \end{vmatrix} - 8\begin{vmatrix} 2 & -1 \\ -7 & -10 \end{vmatrix} + 9\begin{vmatrix} 2 & 0 \\ -7 & 8 \end{vmatrix}$$

$$= 1[0(-10) - (-1)(8)]$$
$$- 8[2(-10) - (-1)(-7)]$$
$$+ 9[2(8) - 0(-7)]$$
$$= 1[0 + 8] - 8[-20 - 7] + 9[16 - 0]$$
$$= 8 - 8[-27] + 9[16]$$
$$= 8 + 216 + 144$$

$\mathbf{\Delta = 368}$

Question 7: Singular and Non-singular Matrices

i. Singular Matrix

A square matrix A is called a singular matrix if its determinant is equal to zero, i.e., $|A| = 0$. A singular matrix does not have a multiplicative inverse.

Example:

Let $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

$|A| = (2 \times 2) - (4 \times 1) = 4 - 4 = 0$

Since $|A| = 0$, A is a **singular matrix**.

ii. Non-singular Matrix

A square matrix A is called a non-singular matrix if its determinant is not equal to zero, i.e., $|A| \neq 0$. Every non-singular matrix has a multiplicative inverse.

Example:

Let $B = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

$|B| = (3 \times 2) - (1 \times 1) = 6 - 1 = 5$

Since $|B| \neq 0$, B is a **non-singular matrix**.

Question 8: Find A^{-1}

Given $A = \begin{bmatrix} 1 & 4 & 2 \\ 7 & 0 & 9 \\ 0 & 2 & -3 \end{bmatrix}$

1. Determinant $|A|$: Expanding along R_1:

$$|A| = 1[0 - 18] - 4[-21 - 0] + 2[14 - 0]$$
$$= -18 + 84 + 28 = 94$$

2. Adjoint of A:

Find cofactors $C_{ij} = (-1)^{i+j} M_{ij}$:

$C_{11} = -18, C_{12} = 21, C_{13} = 14$

$C_{21} = 16, C_{22} = -3, C_{23} = -2$

$C_{31} = 36, C_{32} = 5, C_{33} = -28$

The Adjoint is the transpose of the cofactor matrix:

$$\text{adj}(A) = \begin{bmatrix} -18 & 16 & 36 \\ 21 & -3 & 5 \\ 14 & -2 & -28 \end{bmatrix}$$

3. Multiplicative Inverse:

Using $A^{-1} = \frac{1}{|A|}\text{adj}(A)$:

$$A^{-1} = \frac{1}{94}\begin{bmatrix} -18 & 16 & 36 \\ 21 & -3 & 5 \\ 14 & -2 & -28 \end{bmatrix}$$

Question 9: System of Equations

Solve: $2x + 5y = 27$ and $7x + y = 12$

i) Matrix Inversion Method

Matrix form $AX = B$: $\begin{bmatrix} 2 & 5 \\ 7 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 27 \\ 12 \end{bmatrix}$

1. $|A| = 2(1) - 5(7) = -33$ 2. $\text{adj}(A) = \begin{bmatrix} 1 & -5 \\ -7 & 2 \end{bmatrix}$ 3. $X = A^{-1}B = \frac{1}{-33}\begin{bmatrix} 1 & -5 \\ -7 & 2 \end{bmatrix}\begin{bmatrix} 27 \\ 12 \end{bmatrix}$ $X = \frac{1}{-33}\begin{bmatrix} -33 \\ -165 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ **Solution:** $x = 1, y = 5$

ii) Cramer's Rule

$|A| = -33$

$|A_x| = \begin{vmatrix} 27 & 5 \\ 12 & 1 \end{vmatrix} = 27 - 60 = -33$

$|A_y| = \begin{vmatrix} 2 & 27 \\ 7 & 12 \end{vmatrix} = 24 - 189 = -165$

$x = \frac{|A_x|}{|A|} = \frac{-33}{-33} = 1$

$y = \frac{|A_y|}{|A|} = \frac{-165}{-33} = 5$

Solution: $x = 1, y = 5$