بسم الله الرحمن الرحيم

Solutions of

UNIT#18

Exercise 18.6

Class 10 Math Sindh Board

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Question 1

Given:

$$p : q = r : s$$

This means:

$$\frac{p}{q} = \frac{r}{s} = k$$

for some constant k.

So:

$$p = kq, \quad r = ks$$

We will use this in every part.

1(i)

$$\frac{8p - 3q}{8p + 3q} = \frac{8r - 3s}{8r + 3s}$$

Use $p = kq,\ r = ks$.

Left side:

$$\frac{8p - 3q}{8p + 3q} = \frac{8(kq) - 3q}{8(kq) + 3q} = \frac{q(8k - 3)}{q(8k + 3)} = \frac{8k - 3}{8k + 3}$$

Right side:

$$\frac{8r - 3s}{8r + 3s} = \frac{8(ks) - 3s}{8(ks) + 3s} = \frac{s(8k - 3)}{s(8k + 3)} = \frac{8k - 3}{8k + 3}$$

Both sides are equal, so proved.

1(ii)

$$\sqrt[3]{\frac{p^3 + r^3}{q^3 + s^3}} = \frac{p}{q}$$

Again $p = kq,\ r = ks$.

First simplify the fraction inside the cube root:

$$\frac{p^3 + r^3}{q^3 + s^3} = \frac{(kq)^3 + (ks)^3}{q^3 + s^3} = \frac{k^3q^3 + k^3s^3}{q^3 + s^3} = k^3 \cdot \frac{q^3 + s^3}{q^3 + s^3} = k^3$$

So:

$$\sqrt[3]{\frac{p^3 + r^3}{q^3 + s^3}} = \sqrt[3]{k^3} = k$$

But

$$k = \frac{p}{q}$$

So:

$$\sqrt[3]{\frac{p^3 + r^3}{q^3 + s^3}} = \frac{p}{q}$$

proved.

1(iii)

$$(p^2 + q^2) : \frac{p^3}{p + q} = (r^2 + s^2) : \frac{r^3}{r + s}$$

This ratio means:

$$\frac{p^2 + q^2}{\frac{p^3}{p + q}} = \frac{r^2 + s^2}{\frac{r^3}{r + s}}$$

Simplify left side using $p = kq$.

$$\frac{p^2 + q^2}{\frac{p^3}{p + q}} = (p^2 + q^2) \cdot \frac{p + q}{p^3}$$

Now substitute $p = kq$:

$$p^2 = k^2q^2, \quad p + q = kq + q = (k + 1)q, \quad p^3 = k^3q^3$$

So

$$p^2 + q^2 = k^2q^2 + q^2 = (k^2 + 1)q^2$$

Then:

$$\text{LHS} = (k^2 + 1)q^2 \cdot \frac{(k + 1)q}{k^3q^3} = \frac{(k^2 + 1)(k + 1)q^3}{k^3q^3} = \frac{(k^2 + 1)(k + 1)}{k^3}$$

Now the right side using $r = ks$:

$$\frac{r^2 + s^2}{\frac{r^3}{r + s}} = (r^2 + s^2) \cdot \frac{r + s}{r^3}$$

With $r = ks$:

$$r^2 = k^2s^2, \quad r + s = ks + s = (k+1)s, \quad r^3 = k^3s^3$$
$$r^2 + s^2 = k^2s^2 + s^2 = (k^2+1)s^2$$

So:

$$\text{RHS} = (k^2+1)s^2 \cdot \frac{(k+1)s}{k^3s^3} = \frac{(k^2+1)(k+1)s^3}{k^3s^3} = \frac{(k^2+1)(k+1)}{k^3}$$

LHS = RHS, so proved.

1(iv)

$$p^5 + r^5 : q^5 + s^5 = p^3r^2 : q^3s^2$$

This means:

$$\frac{p^5 + r^5}{q^5 + s^5} = \frac{p^3r^2}{q^3s^2}$$

Use $p = kq,\ r = ks$.

Left side:

$$\frac{p^5 + r^5}{q^5 + s^5} = \frac{(kq)^5 + (ks)^5}{q^5 + s^5} = \frac{k^5q^5 + k^5s^5}{q^5 + s^5} = k^5 \cdot \frac{q^5 + s^5}{q^5 + s^5} = k^5$$

Right side:

$$\frac{p^3r^2}{q^3s^2} = \frac{(kq)^3(ks)^2}{q^3s^2} = \frac{k^3q^3 \cdot k^2s^2}{q^3s^2} = k^5 \cdot \frac{q^3s^2}{q^3s^2} = k^5$$

Both sides equal k^5, so the proportion is true.

1(v)

$$\frac{p-q}{p} : \frac{q}{p+q} = \frac{r-s}{r} : \frac{s}{r+s}$$

Write each ratio as a fraction of fractions:

$$\frac{\frac{p-q}{p}}{\frac{q}{p+q}} = \frac{\frac{r-s}{r}}{\frac{s}{r+s}}$$

Simplify left side:

$$\frac{\frac{p-q}{p}}{\frac{q}{p+q}} = \frac{p-q}{p} \cdot \frac{p+q}{q} = \frac{(p-q)(p+q)}{pq} = \frac{p^2-q^2}{pq}$$

Right side:

$$\frac{\frac{r-s}{r}}{\frac{s}{r+s}} = \frac{r-s}{r} \cdot \frac{r+s}{s} = \frac{(r-s)(r+s)}{rs} = \frac{r^2-s^2}{rs}$$

Now use $p = kq,\ r = ks$.

Left side:

$$\frac{p^2-q^2}{pq} = \frac{(kq)^2-q^2}{kq \cdot q} = \frac{k^2q^2-q^2}{kq^2} = \frac{q^2(k^2-1)}{kq^2} = \frac{k^2-1}{k}$$

Right side:

$$\frac{r^2-s^2}{rs} = \frac{(ks)^2-s^2}{ks \cdot s} = \frac{k^2s^2-s^2}{ks^2} = \frac{s^2(k^2-1)}{ks^2} = \frac{k^2-1}{k}$$

Both sides are equal, so the required proportion is proved.

Question 2

Given:

$$a : b = c : d = e : f$$

So

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$$

Hence we can write:

$$a = kb, \quad c = kd, \quad e = kf$$

We will use this form.

2(i)

$$\frac{a^4b^2 + a^2e^2 - e^4f}{b^6 + b^2f^2 - f^5} = \frac{a^4}{b^4}$$

First simplify the left side using $a = kb,\ e = kf$.

Numerator:

$$a^4b^2 + a^2e^2 - e^4f$$

Substitute:

$$a^4 = k^4b^4, \quad a^2 = k^2b^2, \quad e^2 = k^2f^2, \quad e^4 = k^4f^4$$

So:

$$a^4b^2 = k^4b^4 \cdot b^2 = k^4b^6$$

$$a^2e^2 = (k^2b^2)(k^2f^2) = k^4b^2f^2$$

$$e^4f = k^4f^4 \cdot f = k^4f^5$$

Therefore numerator:

$$= k^4b^6 + k^4b^2f^2 - k^4f^5 = k^4(b^6 + b^2f^2 - f^5)$$

Denominator:

$$b^6 + b^2f^2 - f^5$$

So:

$$\frac{a^4b^2 + a^2e^2 - e^4f}{b^6 + b^2f^2 - f^5} = \frac{k^4(b^6 + b^2f^2 - f^5)}{b^6 + b^2f^2 - f^5} = k^4$$

Now the right side:

$$\frac{a^4}{b^4} = \frac{(kb)^4}{b^4} = \frac{k^4b^4}{b^4} = k^4$$

Both sides are equal to k^4, so the equality is proved.

2(ii)

$$\frac{a^2b + c^2d + e^2f}{ab^2 + cd^2 + ef^2} = \frac{a + c + e}{b + d + f}$$

Use $a = kb,\ c = kd,\ e = kf$.

First the numerator:

$$a^2b = (kb)^2b = k^2b^2 \cdot b = k^2b^3$$

$$c^2d = (kd)^2d = k^2d^2 \cdot d = k^2d^3$$

$$e^2f = (kf)^2f = k^2f^2 \cdot f = k^2f^3$$

So numerator:

$$= k^2(b^3 + d^3 + f^3)$$

Denominator:

$$ab^2 = (kb)b^2 = kb^3$$

$$cd^2 = (kd)d^2 = kd^3$$

$$ef^2 = (kf)f^2 = kf^3$$

So denominator:

$$= k(b^3 + d^3 + f^3)$$

Therefore:

$$\frac{a^2b + c^2d + e^2f}{ab^2 + cd^2 + ef^2} = \frac{k^2(b^3 + d^3 + f^3)}{k(b^3 + d^3 + f^3)} = k$$

Now look at the right side:

$$a + c + e = kb + kd + kf = k(b + d + f)$$

So:

$$\frac{a + c + e}{b + d + f} = \frac{k(b + d + f)}{b + d + f} = k$$

LHS and RHS both equal k. Hence proved.

2(iii)

$$\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$$

Again use $a = kb,\ c = kd,\ e = kf$.

Left side:

$$\frac{ac}{bd} = \frac{(kb)(kd)}{bd} = \frac{k^2bd}{bd} = k^2$$

$$\frac{ce}{df} = \frac{(kd)(kf)}{df} = \frac{k^2df}{df} = k^2$$

$$\frac{ea}{fb} = \frac{(kf)(kb)}{fb} = \frac{k^2fb}{fb} = k^2$$

So:

$$\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = k^2 + k^2 + k^2 = 3k^2$$

Right side:

$$\frac{a^2}{b^2} = \frac{(kb)^2}{b^2} = \frac{k^2b^2}{b^2} = k^2$$

$$\frac{c^2}{d^2} = \frac{(kd)^2}{d^2} = k^2$$

$$\frac{e^2}{f^2} = \frac{(kf)^2}{f^2} = k^2$$

So:

$$\frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2} = k^2 + k^2 + k^2 = 3k^2$$

LHS = RHS = $3k^2$. So the relation is proved.