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Solutions of

UNIT#18

Exercise 18.5

Class 10 Math Sindh Board

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Question 1

> If y varies directly as x^2 and z and $y = 6$ when $x = 4, z = 9$.
> Write y as a function of x and z and find y when $x = -8, z = 12$.

"Varies directly as x^2 and z" means:

$$y \propto x^2 z \quad \Rightarrow \quad y = kx^2 z$$

where k is a constant.

Using the given values $y = 6,\ x = 4,\ z = 9$:

$$6 = k(4)^2(9)$$

$$6 = k(16)(9) = 144k$$

$$k = \frac{6}{144} = \frac{1}{24}$$

So the formula for y is:

$$\boxed{y = \frac{1}{24}x^2 z}$$

Now find y when $x = -8,\ z = 12$:

$$y = \frac{1}{24}(-8)^2(12)$$

$$(-8)^2 = 64$$

$$y = \frac{1}{24} \times 64 \times 12$$

$$64 \times 12 = 768, \quad \frac{768}{24} = 32$$

So

$$\boxed{y = 32}$$

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Question 2

> If y varies directly as x and u^2 and inversely as v and t.
> $y = 40$ when $x = 8, u = 5, v = 3, t = 2$.
> Find y in terms of x, u, v, t. Also find y when $x = -2, u = 4, v = 3, t = -1$.

"Directly as x and u^2 and inversely as v and t" means:

$$y \propto \frac{xu^2}{vt} \quad \Rightarrow \quad y = k\frac{xu^2}{vt}$$

Use the first set of values to find k:

$$40 = k \cdot \frac{8 \cdot 5^2}{3 \cdot 2}$$

$$5^2 = 25$$

$$40 = k \cdot \frac{8 \cdot 25}{6} = k \cdot \frac{200}{6} = k \cdot \frac{100}{3}$$

So:

$$k = 40 \cdot \frac{3}{100} = \frac{120}{100} = \frac{6}{5}$$

Thus,

$$\boxed{y = \frac{6}{5} \cdot \frac{xu^2}{vt} = \frac{6xu^2}{5vt}}$$

Now use this formula for $x = -2,\ u = 4,\ v = 3,\ t = -1$:

$$y = \frac{6 \cdot (-2) \cdot 4^2}{5 \cdot 3 \cdot (-1)}$$

$$4^2 = 16$$

$$y = \frac{6 \cdot (-2) \cdot 16}{5 \cdot 3 \cdot (-1)} = \frac{-192}{-15} = \frac{192}{15}$$

Divide numerator and denominator by 3:

$$y = \frac{64}{5}$$

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So

$$\boxed{y = \frac{64}{5}}$$

Question 3

> If w varies directly as u^2 and inversely as cube root of v, and $w = 216$ when $u = 6$ and $v = 27$.
> Find w when $u = 10$ and $v = 125$.

"Directly as u^2 and inversely as cube root of v" means:

$$w \propto \frac{u^2}{\sqrt[3]{v}} \quad \Rightarrow \quad w = k\frac{u^2}{\sqrt[3]{v}}$$

Use $w = 216,\ u = 6,\ v = 27$:

$$216 = k \cdot \frac{6^2}{\sqrt[3]{27}}$$

$$6^2 = 36, \quad \sqrt[3]{27} = 3$$

$$216 = k \cdot \frac{36}{3} = k \cdot 12$$

$$k = \frac{216}{12} = 18$$

So the formula is:

$$w = 18 \cdot \frac{u^2}{\sqrt[3]{v}}$$

Now for $u = 10,\ v = 125$:

$$\sqrt[3]{125} = 5$$

$$w = 18 \cdot \frac{10^2}{5} = 18 \cdot \frac{100}{5} = 18 \cdot 20 = 360$$

So

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$$\boxed{w = 360}$$

Question 4

> Time period T of a simple pendulum is directly proportional to $\sqrt{L}$ and inversely proportional to $\sqrt{g}$.
> If $T = 2$ sec when $L = 100$ and $g = 9.8$, find T when $L = 200$ and $g = 7.6$.

From statement:

$$T \propto \frac{\sqrt{L}}{\sqrt{g}} \quad \Rightarrow \quad T = k\frac{\sqrt{L}}{\sqrt{g}}$$

We have two situations:
First: $T_1 = 2,\ L_1 = 100,\ g_1 = 9.8$
Second: $T_2 = ?,\ L_2 = 200,\ g_2 = 7.6$

Since the constant k is same in both, we can write a ratio:

$$\frac{T_2}{T_1} = \frac{\frac{\sqrt{L_2}}{\sqrt{g_2}}}{\frac{\sqrt{L_1}}{\sqrt{g_1}}} = \frac{\sqrt{L_2}}{\sqrt{L_1}} \cdot \frac{\sqrt{g_1}}{\sqrt{g_2}}$$

So:

$$T_2 = T_1 \cdot \frac{\sqrt{L_2}}{\sqrt{L_1}} \cdot \frac{\sqrt{g_1}}{\sqrt{g_2}}$$

Now substitute values:

$$T_2 = 2 \cdot \frac{\sqrt{200}}{\sqrt{100}} \cdot \frac{\sqrt{9.8}}{\sqrt{7.6}}$$

$$\sqrt{200} = \sqrt{2 \cdot 100} = 10\sqrt{2}, \quad \sqrt{100} = 10$$

So:

$$\frac{\sqrt{200}}{\sqrt{100}} = \frac{10\sqrt{2}}{10} = \sqrt{2}$$

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Also:

$$\frac{\sqrt{9.8}}{\sqrt{7.6}} = \sqrt{\frac{9.8}{7.6}} = \sqrt{\frac{98}{76}} = \sqrt{\frac{49}{19}} = \frac{7}{\sqrt{19}}$$

Therefore:

$$T_2 = 2 \cdot \sqrt{2} \cdot \frac{7}{\sqrt{19}} = \frac{14\sqrt{2}}{\sqrt{19}} = \frac{14}{\sqrt{19/2}}$$

But it is nicer to simplify like this:

$$2 \cdot \sqrt{2} \cdot \frac{7}{\sqrt{19}} = 2 \cdot \frac{7\sqrt{2}}{\sqrt{19}} = \frac{14\sqrt{2}}{\sqrt{19}}$$

Or using earlier shortcut (combining inside one root):

Actually, combining directly:

$$T_2 = 2 \cdot \sqrt{\frac{200}{100}} \cdot \sqrt{\frac{9.8}{7.6}} = 2 \cdot \sqrt{2} \cdot \sqrt{\frac{49}{38}} = 2 \cdot \sqrt{\frac{98}{38}} = 2 \cdot \sqrt{\frac{49}{19}} = 2 \cdot \frac{7}{\sqrt{19}} = \frac{14}{\sqrt{19}}$$

So the exact answer is:

$$\boxed{T = \frac{14}{\sqrt{19}} \text{ seconds}}$$

Approximate value (for students):

$$\sqrt{19} \approx 4.36$$

$$T \approx \frac{14}{4.36} \approx 3.2 \text{ seconds}$$

Question 5

> Volume V of a gas is directly proportional to temperature T and inversely proportional to $\sqrt{P}$, where P is pressure.
> If $V = 100$ when $T = 30$ and $P = 64$, find V when $T = 60$ and $P = 81$.

"Directly as T and inversely as $\sqrt{P}$" means:

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$$V \propto \frac{T}{\sqrt{P}} \quad \Rightarrow \quad V = k\frac{T}{\sqrt{P}}$$

Use $V = 100,\ T = 30,\ P = 64$:

$$100 = k \cdot \frac{30}{\sqrt{64}}$$

$$\sqrt{64} = 8$$

$$100 = k \cdot \frac{30}{8} = k \cdot \frac{15}{4}$$

So:

$$k = 100 \cdot \frac{4}{15} = \frac{400}{15} = \frac{80}{3}$$

Thus:

$$V = \frac{80}{3} \cdot \frac{T}{\sqrt{P}} = \frac{80T}{3\sqrt{P}}$$

Now find V when $T = 60,\ P = 81$:

$$V = \frac{80 \cdot 60}{3\sqrt{81}}$$

$$\sqrt{81} = 9$$

$$V = \frac{4800}{3 \cdot 9} = \frac{4800}{27}$$

Divide numerator and denominator by 3:

$$V = \frac{1600}{9}$$

So

$$\boxed{V = \frac{1600}{9}}$$

(If needed as decimal, $\frac{1600}{9} \approx 177.8$.)

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