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Solutions of

UNIT#18

Exercise 18.4

Class 10 Math Sindh Board

NotesOfMath.com

Solved Exercises, Past Papers
MCQ Tests, Imprtant Formulas

Question 1

Given:

$$\frac{x}{y} = \frac{z}{w}$$

So there is a common constant k such that

$$\frac{x}{y} = \frac{z}{w} = k$$

which means

$$x = ky, \quad z = kw$$

We will use this in every part.

1(i) $\dfrac{4x + 3y}{4x - 3y} = \dfrac{4z + 3w}{4z - 3w}$

Put $x = ky,\ z = kw.$

Left side:

$$\frac{4x + 3y}{4x - 3y} = \frac{4(ky) + 3y}{4(ky) - 3y} = \frac{y(4k + 3)}{y(4k - 3)} = \frac{4k + 3}{4k - 3}$$

Right side:

$$\frac{4z + 3w}{4z - 3w} = \frac{4(kw) + 3w}{4(kw) - 3w} = \frac{w(4k + 3)}{w(4k - 3)} = \frac{4k + 3}{4k - 3}$$

Both sides are the same, so proved.

1(ii) $\dfrac{5x - 3y}{5x + 3y} = \dfrac{5z - 3w}{5z + 3w}$

Again, $x = ky,\ z = kw.$

Left side:

$$\frac{5x - 3y}{5x + 3y} = \frac{5(ky) - 3y}{5(ky) + 3y} = \frac{y(5k - 3)}{y(5k + 3)} = \frac{5k - 3}{5k + 3}$$

Right side:

$$\frac{5z - 3w}{5z + 3w} = \frac{5(kw) - 3w}{5(kw) + 3w} = \frac{w(5k - 3)}{w(5k + 3)} = \frac{5k - 3}{5k + 3}$$

So both sides are equal. ✔

1(iii) $\dfrac{x^3+y^3}{x^3-y^3} = \dfrac{z^3+w^3}{z^3-w^3}$

Use $x = ky,\ z = kw.$

Left side:

$$\frac{x^3+y^3}{x^3-y^3} = \frac{(ky)^3+y^3}{(ky)^3-y^3} = \frac{k^3y^3+y^3}{k^3y^3-y^3} = \frac{y^3(k^3+1)}{y^3(k^3-1)} = \frac{k^3+1}{k^3-1}$$

Right side:

$$\frac{z^3+w^3}{z^3-w^3} = \frac{(kw)^3+w^3}{(kw)^3-w^3} = \frac{k^3w^3+w^3}{k^3w^3-w^3} = \frac{w^3(k^3+1)}{w^3(k^3-1)} = \frac{k^3+1}{k^3-1}$$

Equal ⇒ proved. ✔

1(iv) $\dfrac{3x+2y}{3x-2y} = \dfrac{3z+2w}{3z-2w}$

Again, $x = ky,\ z = kw.$

Left side:

$$\frac{3x+2y}{3x-2y} = \frac{3(ky)+2y}{3(ky)-2y} = \frac{y(3k+2)}{y(3k-2)} = \frac{3k+2}{3k-2}$$

Right side:

$$\frac{3z+2w}{3z-2w} = \frac{3(kw)+2w}{3(kw)-2w} = \frac{w(3k+2)}{w(3k-2)} = \frac{3k+2}{3k-2}$$

So both sides are equal.

1(v)

$$\frac{2x+3y+2z+3w}{2x+3y-2z-3w} = \frac{2x-3y+2z-3w}{2x-3y-2z+3w}$$

Put $x = ky,\ z = kw.$

Numerator of LHS:

$$2x+3y+2z+3w = 2ky+3y+2kw+3w = y(2k+3)+w(2k+3) = (2k+3)(y+w)$$

Denominator of LHS:

$$2x + 3y - 2z - 3w = 2ky + 3y - 2kw - 3w = y(2k + 3) - w(2k + 3) = (2k + 3)(y - w)$$

So

$$\text{LHS} = \frac{(2k + 3)(y + w)}{(2k + 3)(y - w)} = \frac{y + w}{y - w}$$

Now RHS.

Numerator of RHS:

$$2x - 3y + 2z - 3w = 2ky - 3y + 2kw - 3w = y(2k - 3) + w(2k - 3) = (2k - 3)(y + w)$$

Denominator of RHS:

$$2x - 3y - 2z + 3w = 2ky - 3y - 2kw + 3w = y(2k - 3) - w(2k - 3) = (2k - 3)(y - w)$$

So

$$\text{RHS} = \frac{(2k - 3)(y + w)}{(2k - 3)(y - w)} = \frac{y + w}{y - w}$$

LHS = RHS, so proved. ✅

◆ Question 2

Assume $a : b = c : d$.
So

$$\frac{a}{b} = \frac{c}{d} = k \quad \Rightarrow \quad a = kb,\ c = kd$$

2(i)

$$\frac{a^2 - b^2}{a^2 + b^2} : \frac{a^2 + b^2}{a^2 - b^2} = \frac{ac - bd}{ac + bd} : \frac{ac + bd}{ac - bd}$$

First show:

$$\frac{a^2 - b^2}{a^2 + b^2} = \frac{ac - bd}{ac + bd}$$

Using $a = kb,\ c = kd$:

$$a^2 = k^2b^2, \quad ac = (kb)(kd) = k^2bd$$

Then

$$\frac{a^2-b^2}{a^2+b^2}=\frac{k^2b^2-b^2}{k^2b^2+b^2}=\frac{b^2(k^2-1)}{b^2(k^2+1)}=\frac{k^2-1}{k^2+1}$$

Also

$$\frac{ac-bd}{ac+bd}=\frac{k^2bd-bd}{k^2bd+bd}=\frac{bd(k^2-1)}{bd(k^2+1)}=\frac{k^2-1}{k^2+1}$$

So

$$\frac{a^2-b^2}{a^2+b^2}=\frac{ac-bd}{ac+bd}$$

Taking reciprocal on both sides:

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

Therefore the two ratios written with ":" are equal, as required.

2(ii)

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

Again, $a=kb,\ c=kd.$

Left side:

$$a+b+c+d=kb+b+kd+d=(k+1)(b+d)$$

$$a+b-c-d=kb+b-kd-d=(k+1)(b-d)$$

So

$$\text{LHS}=\frac{(k+1)(b+d)}{(k+1)(b-d)}=\frac{b+d}{b-d}$$

Right side:

$$a-b+c-d=kb-b+kd-d=(k-1)(b+d)$$

$$a-b-c+d=kb-b-kd+d=(k-1)(b-d)$$

So

$$\text{RHS}=\frac{(k-1)(b+d)}{(k-1)(b-d)}=\frac{b+d}{b-d}$$

Hence LHS = RHS, so proved.

Question 3 — Using Componendo & Dividendo

Theorem (Componendo–Dividendo):

If

$$\frac{A}{B}=\frac{C}{D},$$

then

$$\frac{A+B}{A-B}=\frac{C+D}{C-D}$$

(provided denominators are not zero).

We will apply it cleverly in each part.

3(i)

$$\frac{(x+3)^2-(x-5)^2}{(x+3)^2+(x-5)^2}=\frac{4}{5}$$

Let

$$A=(x+3)^2,\quad B=(x-5)^2.$$

So equation is:

$$\frac{A-B}{A+B}=\frac{4}{5}$$

Apply componendo and dividendo to the ratio $\dfrac{A-B}{A+B}$:

$$\frac{(A-B)+(A+B)}{(A-B)-(A+B)}=\frac{4+5}{4-5}$$

Left side:

$$\frac{2A}{-2B}=-\frac{A}{B}$$

Right side:

$$\frac{9}{-1}=-9$$

So

$$-\frac{A}{B} = -9 \quad \Rightarrow \quad \frac{A}{B} = 9$$

That is:

$$\frac{(x+3)^2}{(x-5)^2} = 9$$

Take square root (remember ±):

$$\frac{x+3}{x-5} = \pm 3$$

Case 1: $\frac{x+3}{x-5} = 3$

$$x + 3 = 3(x - 5)$$

$$x + 3 = 3x - 15$$

$$3 + 15 = 3x - x$$

$$18 = 2x \Rightarrow x = 9$$

Case 2: $\frac{x+3}{x-5} = -3$

$$x + 3 = -3(x - 5)$$

$$x + 3 = -3x + 15$$

$$x + 3x = 15 - 3$$

$$4x = 12 \Rightarrow x = 3$$

So,

$$\boxed{x = 3,\ 9}$$

3(ii)

$$\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = \frac{1}{2}$$

Let

$$A = \sqrt{x+1}, \quad B = \sqrt{x-1}.$$

So:

$$\frac{A+B}{A-B} = \frac{1}{2}$$

Apply componendo and dividendo to $\frac{A+B}{A-B}$:

$$\frac{(A+B)+(A-B)}{(A+B)-(A-B)} = \frac{1+2}{1-2}$$

Left:

$$\frac{2A}{2B} = \frac{A}{B}$$

Right:

$$\frac{3}{-1} = -3$$

So

$$\frac{A}{B} = -3$$

i.e.

$$\frac{\sqrt{x+1}}{\sqrt{x-1}} = -3$$

But $\sqrt{x+1} \geq 0$ and $\sqrt{x-1} \geq 0$ for real x.
So the left side is **non-negative**, but the right side is -3 (negative).

Therefore there is **no real value of** x satisfying the equation.

$$\boxed{\text{No real solution}}$$

3(iii)

$$\frac{\sqrt{x+5} - \sqrt{x-5}}{\sqrt{x+5} + \sqrt{x-5}} = \frac{1}{10}$$

Let

$$A = \sqrt{x+5}, \quad B = \sqrt{x-5}.$$

Equation:

$$\frac{A-B}{A+B} = \frac{1}{10}$$

Apply componendo and dividendo to $\frac{A-B}{A+B}$:

$$\frac{(A-B)+(A+B)}{(A-B)-(A+B)} = \frac{1+10}{1-10}$$

Left:

$$\frac{2A}{-2B} = -\frac{A}{B}$$

Right:

$$\frac{11}{-9} = -\frac{11}{9}$$

So

$$-\frac{A}{B} = -\frac{11}{9} \quad \Rightarrow \quad \frac{A}{B} = \frac{11}{9}$$

That is:

$$\frac{\sqrt{x+5}}{\sqrt{x-5}} = \frac{11}{9}$$

Square both sides:

$$\frac{x+5}{x-5} = (\frac{11}{9})^2 = \frac{121}{81}$$

Now solve:

$$\frac{x+5}{x-5} = \frac{121}{81}$$

Cross multiply:

$$81(x+5) = 121(x-5)$$

$$81x + 405 = 121x - 605$$

Bring x-terms and numbers together:

$$405 + 605 = 121x - 81x$$

$$1010 = 40x$$

$$x = \frac{1010}{40} = \frac{101}{4}$$

So,

$$\boxed{x = \frac{101}{4}}$$

(Domain check: $x > 5$, and $\frac{101}{4} = 25.25 > 5$, so it is valid.)

3(iv)

$$\frac{(x-4)^3 - (x-3)^3}{(x-4)^3 + (x-3)^3} = \frac{63}{65}$$

Let

$$A = (x-4)^3, \quad B = (x-3)^3.$$

Equation:

$$\frac{A-B}{A+B} = \frac{63}{65}$$

Apply componendo and dividendo to $\dfrac{A-B}{A+B}$:

$$\frac{(A-B)+(A+B)}{(A-B)-(A+B)} = \frac{63+65}{63-65}$$

Left:

$$\frac{2A}{-2B} = -\frac{A}{B}$$

Right:

$$\frac{128}{-2} = -64$$

So:

$$-\frac{A}{B} = -64 \quad \Rightarrow \quad \frac{A}{B} = 64$$

That is:

$$\frac{(x-4)^3}{(x-3)^3} = 64 = 4^3$$

Take cube root on both sides:

$$\frac{x-4}{x-3} = 4$$

Now solve:

$$x - 4 = 4(x - 3)$$

$$x - 4 = 4x - 12$$

$$-4 + 12 = 4x - x$$

$$8 = 3x$$

$$x = \frac{8}{3}$$

So,

$$\boxed{x = \frac{8}{3}}$$