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Solutions of

UNIT#18

Exercise 18.3

Class 10 Math Sindh Board

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MCQ Tests, Imprtant Formulas

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1. Find the third proportional if first, second and fourth proportional are

Formula:

If first = A, second = B, third = T, fourth = D, then

$A : B = T : D \Rightarrow \frac{A}{B} = \frac{T}{D}$ so $T = \frac{A \cdot D}{B}$.

(i) $6,\ 18$ and 54

Let third proportional be T.

$$\frac{6}{18} = \frac{T}{54}$$

$$6 \times 54 = 18T$$

$$18T = 324 \Rightarrow T = \frac{324}{18} = 18$$

Answer: 18

(ii) $a^2 - b^2,\ a + b$ and $a - b$

Let third proportional be T.

$$\frac{a^2 - b^2}{a + b} = \frac{T}{a - b}$$

$$(a^2 - b^2)(a - b) = (a + b)T$$

But $a^2 - b^2 = (a - b)(a + b)$, so

$$(a - b)(a + b)(a - b) = (a + b)T$$

Cancel $(a + b)$:

$$(a - b)^2 = T$$

Answer: $(a - b)^2$

(iii) $(x + y)^2,\ x^3 + y^3$ and $x + y$

Let third proportional be T.

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$$\frac{(x+y)^2}{x^3+y^3} = \frac{T}{x+y}$$

$$(x+y)^2(x+y) = T(x^3+y^3)$$

$$(x+y)^3 = T(x^3+y^3)$$

Use $x^3+y^3 = (x+y)(x^2-xy+y^2)$:

$$(x+y)^3 = T(x+y)(x^2-xy+y^2)$$

Cancel $x+y$:

$$(x+y)^2 = T(x^2-xy+y^2)$$

$$T = \frac{(x+y)^2}{x^2-xy+y^2}$$

Answer: $\frac{(x+y)^2}{x^2-xy+y^2}$

(iv) $\frac{a^3+b^3}{a^2-b^2}$, $\frac{a^2-ab+b^2}{a-b}$ **and** $a+b$

Let third proportional be T.

$$\frac{\frac{a^3+b^3}{a^2-b^2}}{\frac{a^2-ab+b^2}{a-b}} = \frac{T}{a+b}$$

This is

$$\frac{a^3+b^3}{a^2-b^2} \times \frac{a-b}{a^2-ab+b^2} = \frac{T}{a+b}$$

Use the factors

$$a^3+b^3 = (a+b)(a^2-ab+b^2), \quad a^2-b^2 = (a-b)(a+b)$$

So LHS becomes

$$\frac{(a+b)(a^2-ab+b^2)}{(a-b)(a+b)} \times \frac{a-b}{a^2-ab+b^2} = 1$$

So

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$$1 = \frac{T}{a+b} \Rightarrow T = a+b$$

Answer: $a+b$

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2. Find the fourth proportional to

Formula:

If first = A, second = B, third = C, fourth = D, then

$A : B = C : D \Rightarrow \frac{A}{B} = \frac{C}{D}$ so $D = \frac{B \cdot C}{A}$.

(i) $8, 4, 2$

$$D = \frac{4 \times 2}{8} = \frac{8}{8} = 1$$

Answer: 1

(ii) $a^3 + b^3,\ a^2 - b^2,\ a^2 - ab + b^2$

$$D = \frac{(a^2 - b^2)(a^2 - ab + b^2)}{a^3 + b^3}$$

Use $a^2 - b^2 = (a-b)(a+b)$ and $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$:

$$D = \frac{(a-b)(a+b)(a^2 - ab + b^2)}{(a+b)(a^2 - ab + b^2)} = a - b$$

Answer: $a - b$

(iii) $a^2 - 8a + 12,\ a - 2,\ 2a^3 - 12a^2$

First factor:

$$a^2 - 8a + 12 = (a-2)(a-6), \quad 2a^3 - 12a^2 = 2a^2(a-6)$$

Now

$$D = \frac{(a-2)(2a^3 - 12a^2)}{a^2 - 8a + 12} = \frac{(a-2) \cdot 2a^2(a-6)}{(a-2)(a-6)} = 2a^2$$

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Answer: $2a^2$

(iv) $(a^2-b^2)(a^2-ab+b^2),\ a^3+b^3,\ a^3-b^3$

Let

$$A=(a^2-b^2)(a^2-ab+b^2),\ B=a^3+b^3,\ C=a^3-b^3$$

Then

$$D=\frac{BC}{A}$$

Write them in factors:

$$A=(a-b)(a+b)(a^2-ab+b^2)$$

$$B=(a+b)(a^2-ab+b^2),\quad C=(a-b)(a^2+ab+b^2)$$

So

$$D=\frac{(a+b)(a^2-ab+b^2)\,(a-b)(a^2+ab+b^2)}{(a-b)(a+b)(a^2-ab+b^2)}=a^2+ab+b^2$$

Answer: a^2+ab+b^2

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3. Find the mean proportional to

Formula (mean proportional):
Between A and B, let mean be M. Then
$A:M=M:B$, so $M^2=AB, M=\sqrt{AB}$.

(i) $8, 18$

$$M^2=8\times 18=144\Rightarrow M=12$$

Answer: 12

(ii) $5ab^2,\ 20a^3b^2$

$$M^2=5ab^2\times 20a^3b^2=100a^4b^4$$

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$$M = \sqrt{100a^4b^4} = 10a^2b^2$$

Answer: $10a^2b^2$

(iii) $a^4 - b^4,\ \dfrac{a^2 - b^2}{a^2 + b^2}$

$$M^2 = (a^4 - b^4)\left(\frac{a^2 - b^2}{a^2 + b^2}\right)$$

Use $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$:

$$M^2 = (a^2 - b^2)(a^2 + b^2) \cdot \frac{a^2 - b^2}{a^2 + b^2} = (a^2 - b^2)^2$$

$$M = a^2 - b^2$$

Answer: $a^2 - b^2$

(iv) $a^3 - b^3,\ \dfrac{a - b}{a^2 + ab + b^2}$

$$M^2 = (a^3 - b^3)\left(\frac{a - b}{a^2 + ab + b^2}\right)$$

Use $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$:

$$M^2 = (a - b)(a^2 + ab + b^2) \cdot \frac{a - b}{a^2 + ab + b^2} = (a - b)^2$$

$$M = a - b$$

Answer: $a - b$

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4. Find the value of x in the following continued proportions

Continued proportion:

If a, b, c are in continued proportion, then $a : b = b : c$.

So $\dfrac{a}{b} = \dfrac{b}{c} \Rightarrow b^2 = ac.$

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(i) $45, x, 5$

$$\frac{45}{x} = \frac{x}{5} \Rightarrow x^2 = 45 \times 5 = 225$$

$$x = 15$$

Answer: 15

(ii) $16, x, 9$

$$\frac{16}{x} = \frac{x}{9} \Rightarrow x^2 = 16 \times 9 = 144$$

$$x = 12$$

Answer: 12

(iii) $12, 3x - 6, 27$

$$\frac{12}{3x-6} = \frac{3x-6}{27}$$

$$(3x-6)^2 = 12 \times 27 = 324$$

$$3x - 6 = 18 \quad \text{(take positive value)}$$

$$3x = 24 \Rightarrow x = 8$$

Answer: 8

(iv) $7, x - 3, 112$

$$\frac{7}{x-3} = \frac{x-3}{112}$$

$$(x-3)^2 = 7 \times 112 = 784$$

$$x - 3 = 28 \quad \text{(take positive value)}$$

$$x = 31$$

Answer: 31

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