بسم الله الرحمن الرحيم

Solutions of

UNIT #18

Exercise 18.1

Class 10 Math Sindh Board

NotesOfMath.com

Solved Exercises, Past Papers
MCQ Tests, Imprtant Formulas

www.notesofmath.com

1. 📏 Finding the Ratio of the Following

The key to finding a ratio is ensuring both quantities are in the **same unit** before simplifying.

(i) $70kg$ **and** $28kg$

$$\text{Ratio} = \frac{70\ \text{kg}}{28\ \text{kg}} = \frac{70}{28}$$

Divide both by their greatest common divisor, which is 14:

$$\frac{70 \div 14}{28 \div 14} = \frac{5}{2}$$

Ratio: $5 : 2$

(ii) $60cm$ **and** $1m$

First, convert $1m$ to cm. Since $1m = 100cm$:

$$\text{Ratio} = \frac{60\ \text{cm}}{100\ \text{cm}} = \frac{60}{100}$$

Divide both by 20:

$$\frac{60 \div 20}{100 \div 20} = \frac{3}{5}$$

Ratio: $3 : 5$

(iii) $40sec$, $3min$

First, convert $3min$ to seconds. Since $1min = 60sec$:

$$3\ \text{min} = 3 \times 60\ \text{sec} = 180\ \text{sec}$$

$$\text{Ratio} = \frac{40\ \text{sec}}{180\ \text{sec}} = \frac{40}{180}$$

Divide both by 20:

$$\frac{40 \div 20}{180 \div 20} = \frac{2}{9}$$

Ratio: $2 : 9$

(iv) $200ml$ **and** $2l$

First, convert $2l$ to ml. Since $1l = 1000ml$:

$$2\,\text{l} = 2 \times 1000\,\text{ml} = 2000\,\text{ml}$$

$$\text{Ratio} = \frac{200\,\text{ml}}{2000\,\text{ml}} = \frac{200}{2000}$$

Divide both by 200:

$$\frac{200 \div 200}{2000 \div 200} = \frac{1}{10}$$

Ratio: $1 : 10$

(v) $135°$ **and** $360°$

$$\text{Ratio} = \frac{135°}{360°} = \frac{135}{360}$$

Divide both by their greatest common divisor, which is 45:

$$\frac{135 \div 45}{360 \div 45} = \frac{3}{8}$$

Ratio: $3 : 8$

(vi) $3.5kg$, $5kg\ 200gm$

First, convert both quantities to grams (gm). Since $1kg = 1000gm$:

- $3.5\,\text{kg} = 3.5 \times 1000\,\text{gm} = 3500\,\text{gm}$
- $5\,\text{kg}\ 200\,\text{gm} = 5 \times 1000 + 200\,\text{gm} = 5000 + 200\,\text{gm} = 5200\,\text{gm}$

$$\text{Ratio} = \frac{3500\,\text{gm}}{5200\,\text{gm}} = \frac{3500}{5200} = \frac{35}{52}$$

(The fraction $\frac{35}{52}$ cannot be simplified further as 35 is 5×7 and 52 is $2 \times 2 \times 13$). **Ratio:** $35 : 52$

---

2. 🏭 Factory Workers Ratios

Given Data:

- Total workers $= 120$
- Number of women $= 45$
- Number of men = Total workers − Number of women

$$120 - 45 = 75 \text{ men}$$

(i) men to women

$$\text{Ratio} = \text{Men} : \text{Women} = 75 : 45$$

Divide both by their greatest common divisor, 15:

$$75 \div 15 : 45 \div 15 = 5 : 3$$

Ratio: $5 : 3$

(ii) women to men

$$\text{Ratio} = \text{Women} : \text{Men} = 45 : 75$$

Divide both by 15:

$$45 \div 15 : 75 \div 15 = 3 : 5$$

Ratio: $3 : 5$

(iii) women to total worker

$$\text{Ratio} = \text{Women} : \text{Total Workers} = 45 : 120$$

Divide both by 15:

$$45 \div 15 : 120 \div 15 = 3 : 8$$

Ratio: $3 : 8$

(iv) men to total workers

$$\text{Ratio} = \text{Men} : \text{Total Workers} = 75 : 120$$

Divide both by 15:

$$75 \div 15 : 120 \div 15 = 5 : 8$$

Ratio: $5 : 8$

---

3. 📝 Algebraic Ratio

Problem: If $5(4x - 2y) = 3x - 4y$, find $x : y$.

Step 1: Simplify the equation.

$$20x - 10y = 3x - 4y$$

Step 2: Collect x terms on one side and y terms on the other. Subtract $3x$ from both sides and add $10y$ to both sides:

$$20x - 3x = 10y - 4y$$

$$17x = 6y$$

Step 3: Express the relationship as a ratio $\frac{x}{y}$. Divide both sides by y and then by 17:

$$\frac{x}{y} = \frac{6}{17}$$

Ratio $x : y$ is $6 : 17$

---

4. ⚖️ Equal Ratios (Proportion)

Problem: Find the value of 'a' if the ratios $3a + 4 : 2a + 5$ and $4 : 3$ are equal.

Since the ratios are equal, we can set up a proportion:

$$\frac{3a+4}{2a+5}=\frac{4}{3}$$

Step 1: Cross-multiply.

$$3(3a+4)=4(2a+5)$$

Step 2: Distribute on both sides.

$$9a+12=8a+20$$

Step 3: Solve for a. Subtract $8a$ from both sides:

$$9a-8a+12=20$$

$$a+12=20$$

Subtract 12 from both sides:

$$a=20-12$$

$$a=8$$

The value of a is 8.

---

5. ➕ Adding to a Ratio

Problem: What number must be added to antecedent and consequent of the ratio $5:27$ to make it equal to $1:3$?

Let the number to be added be x.

- Antecedent: 5
- Consequent: 27 The new ratio is $\frac{5+x}{27+x}$, which must equal $\frac{1}{3}$.

$$\frac{5+x}{27+x}=\frac{1}{3}$$

Step 1: Cross-multiply.

$$3(5+x) = 1(27+x)$$

$$15 + 3x = 27 + x$$

Step 2: Solve for x. Subtract x from both sides:

$$15 + 3x - x = 27$$

$$15 + 2x = 27$$

Subtract 15 from both sides:

$$2x = 27 - 15$$

$$2x = 12$$

Divide by 2:

$$x = 6$$

The number that must be added is 6.

---

6. 🧮 Substitution into a Ratio

Problem: If $a : b = 5 : 8$, find the value of $3a + 4b : 5a + 7b$.

Since $a : b = 5 : 8$, we can write $a = 5k$ and $b = 8k$ for some constant $k \neq 0$. Alternatively, we can divide the expression by b:

$$\text{Required Ratio} = \frac{3a + 4b}{5a + 7b}$$

Divide the numerator and denominator by b:

$$\frac{\frac{3a}{b} + \frac{4b}{b}}{\frac{5a}{b} + \frac{7b}{b}} = \frac{3\left(\frac{a}{b}\right) + 4}{5\left(\frac{a}{b}\right) + 7}$$

Substitute the given value $\frac{a}{b} = \frac{5}{8}$:

$$\frac{3\left(\frac{5}{8}\right)+4}{5\left(\frac{5}{8}\right)+7}=\frac{\frac{15}{8}+4}{\frac{25}{8}+7}$$

Find a common denominator (8) for the numerator and the denominator separately:

$$\frac{\frac{15}{8}+\frac{4\times8}{8}}{\frac{25}{8}+\frac{7\times8}{8}}=\frac{\frac{15+32}{8}}{\frac{25+56}{8}}$$

$$\frac{\frac{47}{8}}{\frac{81}{8}}$$

Since $\frac{81}{8}$ is in the denominator, you multiply by its reciprocal $\frac{8}{81}$:

$$\frac{47}{8}\times\frac{8}{81}=\frac{47}{81}$$

The value of the ratio $3a+4b:5a+7b$ is $47:81$.

---

7. ❌ Finding the Value of x

These problems involve **proportion**, where the product of the **extremes** (first and fourth terms) equals the product of the **means** (second and third terms). If $A:B::C:D$, then $A\times D=B\times C$.

(i) $2x+5:5::3x-2:7$

$$\text{Extremes}=(2x+5)\times7$$

$$\text{Means}=5\times(3x-2)$$

$$(2x+5)\times7=5\times(3x-2)$$

$$14x+35=15x-10$$

Subtract $14x$ from both sides:

$$35=15x-14x-10$$

$$35 = x - 10$$

Add 10 to both sides:

$$x = 35 + 10$$

$$x = 45$$

(ii) $\frac{4x-3}{5} : \frac{3}{4} :: \frac{4x}{3} : \frac{7}{2}$

$$\text{Extremes} = (\frac{4x-3}{5}) \times (\frac{7}{2})$$

$$\text{Means} = (\frac{3}{4}) \times (\frac{4x}{3})$$

$$(\frac{4x-3}{5}) \times (\frac{7}{2}) = (\frac{3}{4}) \times (\frac{4x}{3})$$

Simplify the Means side:

$$\frac{3}{4} \times \frac{4x}{3} = x$$

Simplify the Extremes side:

$$\frac{7(4x-3)}{10} = x$$

$$7(4x-3) = 10x$$

$$28x - 21 = 10x$$

Subtract $10x$ from both sides:

$$28x - 10x - 21 = 0$$

$$18x - 21 = 0$$

$$18x = 21$$

$$x = \frac{21}{18}$$

$$x = \frac{7}{6}$$

(iii) $\frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$

$$\text{Extremes} = (\frac{x-3}{2}) \times (\frac{4}{x+4})$$

$$\text{Means} = (\frac{5}{x-1}) \times (\frac{x-1}{3})$$

Equate Extremes and Means:

$$(\frac{x-3}{2}) \times (\frac{4}{x+4}) = (\frac{5}{x-1}) \times (\frac{x-1}{3})$$

Simplify both sides:

$$\frac{4(x-3)}{2(x+4)} = \frac{5(x-1)}{3(x-1)}$$

Assuming $x \neq 1$, we can cancel $(x-1)$ on the right side:

$$\frac{2(x-3)}{x+4} = \frac{5}{3}$$

Cross-multiply:

$$3 \times 2(x-3) = 5(x+4)$$

$$6(x-3) = 5(x+4)$$

$$6x - 18 = 5x + 20$$

Subtract $5x$ from both sides:

$$6x - 5x - 18 = 20$$

$$x - 18 = 20$$

Add 18 to both sides:

$$x = 20 + 18$$

$$x = 38$$

(iv) $(a^2 - ab + b^2) : x :: \frac{a^3+b^3}{a-b} : (a+b)^2$

Recall the algebraic identity: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$.

$$\text{Extremes} = (a^2 - ab + b^2) \times (a+b)^2$$

$$\text{Means} = x \times (\frac{a^3 + b^3}{a - b})$$

Equate Extremes and Means:

$$(a^2 - ab + b^2)(a+b)^2 = x(\frac{a^3 + b^3}{a - b})$$

Substitute the identity $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$:

$$(a^2 - ab + b^2)(a+b)^2 = x(\frac{(a+b)(a^2 - ab + b^2)}{a - b})$$

Assuming $a^2 - ab + b^2 \neq 0$ and $a + b \neq 0$, divide both sides by $(a^2 - ab + b^2)$:

$$(a+b)^2 = x(\frac{a+b}{a-b})$$

Divide both sides by $(a+b)$ (assuming $a + b \neq 0$):

$$a+b=x\left(\frac{1}{a-b}\right)$$

$$a+b=\frac{x}{a-b}$$

Solve for x:

$$x=(a+b)(a-b)$$

Recall the difference of squares identity: $(a+b)(a-b)=a^2-b^2$.

$$x=a^2-b^2$$

(v) $11-x:8-x::25-x:16-x$

$$\text{Extremes}=(11-x)\times(16-x)$$

$$\text{Means}=(8-x)\times(25-x)$$

Equate Extremes and Means:

$$(11-x)(16-x)=(8-x)(25-x)$$

Expand both sides:

$$11(16)-11x-16x+x^2=8(25)-8x-25x+x^2$$

$$176-27x+x^2=200-33x+x^2$$

Subtract x^2 from both sides:

$$176-27x=200-33x$$

Add $33x$ to both sides:

$$176-27x+33x=200$$

$$176 + 6x = 200$$

Subtract 176 from both sides:

$$6x = 200 - 176$$

$$6x = 24$$

$$x = \frac{24}{6}$$

$$x = 4$$