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Solutions of

UNIT#17

Exercise 17.5

Class 10 Math Sindh Board

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Exercise 17.5 Solutions

Question 1

Find x and y:

(i) $(x-5, 10) = (11, y-7)$

When two ordered pairs are equal,
their first parts are equal and their second parts are equal.

So,

- First parts:
  $x - 5 = 11 \Rightarrow x = 11 + 5 = 16$
- Second parts:
  $10 = y - 7 \Rightarrow y = 10 + 7 = 17$

Answer: $x = 16,\ y = 17$

(ii) $(5x+8, 5y-4) = (3x+10, 2y+2)$

Again, match first parts and second parts.

First parts:

$$5x + 8 = 3x + 10$$

Bring like terms together:

$$5x - 3x = 10 - 8 \Rightarrow 2x = 2 \Rightarrow x = 1$$

Second parts:

$$5y - 4 = 2y + 2$$

$$5y - 2y = 2 + 4 \Rightarrow 3y = 6 \Rightarrow y = 2$$

Answer: $x = 1,\ y = 2$

(iii) $(2x-3y, 5x+y) = (3, 16)$

So we have two equations:

1. $2x - 3y = 3$
2. $5x + y = 16$

From equation (2):

$$y = 16 - 5x$$

Substitute this in equation (1):

$$2x - 3(16 - 5x) = 3$$

$$2x - 48 + 15x = 3$$

$$17x - 48 = 3$$

$$17x = 51 \Rightarrow x = 3$$

Now find y:

$$y = 16 - 5x = 16 - 5(3) = 16 - 15 = 1$$

Answer: $x = 3,\ y = 1$

Question 2

|P| = 10, |Q| = 15

- $|P \times Q| = 10 \times 15 = 150$
- $|Q \times P| = 15 \times 10 = 150$
- $|P \times P| = 10 \times 10 = 100$
- $|Q \times Q| = 15 \times 15 = 225$

Answer:
P×Q: 150, Q×P: 150, P×P: 100, Q×Q: 225

Question 3

$A = \{1, 2\},\ B = \{2, 3\},\ C = \{1, 3, 5\}$

(i) $A \times B$

Pairs with first from A and second from B:

$$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$$

(ii) $B \times C$

$$B \times C = \{(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)\}$$

(iii) $A \times (B \cup C)$

First find $B \cup C$:

$$B \cup C = \{2, 3\} \cup \{1, 3, 5\} = \{1, 2, 3, 5\}$$

Now:

$$A \times (B \cup C) = \{(1,1),(1,2),(1,3),(1,5),(2,1),(2,2),(2,3),(2,5)\}$$

(iv) $B \times (A \cup C)$

First:

$$A \cup C = \{1,2\} \cup \{1,3,5\} = \{1,2,3,5\}$$

Then:

$$B \times (A \cup C) = \{(2,1),(2,2),(2,3),(2,5),(3,1),(3,2),(3,3),(3,5)\}$$

(v) $(A \cap B) \times (B \cap C)$

$$A \cap B = \{1,2\} \cap \{2,3\} = \{2\}$$

$$B \cap C = \{2,3\} \cap \{1,3,5\} = \{3\}$$

So:

$$(A \cap B) \times (B \cap C) = \{(2,3)\}$$

Question 4

$A = \{5,6\},\ B = \{1,2,3\}$

First:

$$A \times B = \{(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\}$$

(i) Three relations in $A \times B$

A relation is any subset of $A \times B$. Examples:

- $R_1 = \{(5,1),(6,2)\}$
- $R_2 = \{(5,3)\}$
- $R_3 = A \times B = \{(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\}$

(ii) Four relations in $B \times A$

First:

$$B \times A = \{(1,5),(1,6),(2,5),(2,6),(3,5),(3,6)\}$$

Examples of 4 relations (subsets):

- $S_1 = \{(1, 5)\}$
- $S_2 = \{(2, 5), (3, 6)\}$
- $S_3 = \{(1, 6), (2, 6)\}$
- $S_4 = B \times A$ (all 6 pairs)

(iii) Five relations in B (on B)

Here "in B" means relations on B, i.e. subsets of $B \times B$.

$$B \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

Examples:

- $T_1 = \emptyset$ (empty relation)
- $T_2 = \{(1, 1)\}$
- $T_3 = \{(1, 2), (2, 3)\}$
- $T_4 = \{(1, 1), (2, 2), (3, 3)\}$
- $T_5 = B \times B$

(iv) All relations in A (on A)

First:

$$A \times A = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$$

All relations are **all subsets** of $A \times A$. There are $2^4 = 16$ relations:

1. $\emptyset$
2. $\{(5, 5)\}$
3. $\{(5, 6)\}$
4. $\{(6, 5)\}$
5. $\{(6, 6)\}$
6. $\{(5, 5), (5, 6)\}$
7. $\{(5, 5), (6, 5)\}$
8. $\{(5, 5), (6, 6)\}$
9. $\{(5, 6), (6, 5)\}$
10. $\{(5, 6), (6, 6)\}$
11. $\{(6, 5), (6, 6)\}$
12. $\{(5, 5), (5, 6), (6, 5)\}$
13. $\{(5, 5), (5, 6), (6, 6)\}$
14. $\{(5, 5), (6, 5), (6, 6)\}$

15. $\{(5,6),(6,5),(6,6)\}$
16. $A \times A = \{(5,5),(5,6),(6,5),(6,6)\}$

Question 5

O(A) = 3, O(B) = 4

$$|A \times B| = 3 \times 4 = 12$$

A binary relation is any subset of A×B, so
number of relations = $2^{12} = 4096.$

Answer: 4096 relations.

Question 6

$A = \{0,1,2,3\}, \ B = \{2,4,6,8\}$

We will first list the pairs, then explain table idea.

(i) $R_1 = \{(a,b) | b < 5\}$

In B, numbers less than 5 are 2 and 4.

So for **each** a in A, we pair with 2 and 4:

$$R_1 = \{(0,2),(0,4),(1,2),(1,4),(2,2),(2,4),(3,2),(3,4)\}$$

Tabular idea: rows = A, columns = B, put a "✓" where b is 2 or 4.

(ii) $R_2 = \{(a,b) | a + b = 9\}$

Check each a:

- a = 0 → b = 9 (not in B)
- a = 1 → b = 8 (in B) → (1,8)
- a = 2 → b = 7 (not in B)
- a = 3 → b = 6 (in B) → (3,6)

So:

$$R_2 = \{(1,8),(3,6)\}$$

(iii) $R_3 = \{(a,b) | a - b = 1\}$

So $b = a - 1.$

Check:

- a = 0 → b = -1 (not in B)
- a = 1 → b = 0 (not in B)
- a = 2 → b = 1 (not in B)
- a = 3 → b = 2 (in B) → (3,2)

So:

$$R_3 = \{(3, 2)\}$$

(iv) $R_4 = \{(a, b) | ab = 6\}$

Check products:

- a = 0 → 0×b = 0 (never 6)
- a = 1 → 1×b = 6 → b = 6 (in B) → (1,6)
- a = 2 → 2×b = 6 → b = 3 (not in B)
- a = 3 → 3×b = 6 → b = 2 (in B) → (3,2)

So:

$$R_4 = \{(1, 6), (3, 2)\}$$

Question 7

$R = \{(x, y) | y = 2x + 5\}$ in integers.

(i) Domain is {-2, −1, 0, 1, 2}. Find range.

Compute y:

- x = −2 → y = 2(−2)+5 = −4+5 = 1
- x = −1 → y = 2(−1)+5 = −2+5 = 3
- x = 0 → y = 0+5 = 5
- x = 1 → y = 2+5 = 7
- x = 2 → y = 4+5 = 9

So range = {1, 3, 5, 7, 9}.

(ii) Range is {11, 13, 15, 17}. Find domain.

We have:

$$y = 2x + 5 \Rightarrow x = \frac{y - 5}{2}$$

For each y:

- y = 11 → $x = (11 - 5)/2 = 6/2 = 3$

- y = 13 → $x = (13 - 5)/2 = 8/2 = 4$
- y = 15 → $x = (15 - 5)/2 = 10/2 = 5$
- y = 17 → $x = (17 - 5)/2 = 12/2 = 6$

So domain = {3, 4, 5, 6}.

Question 8

x, y are elements of W (whole numbers: 0,1,2,...)

(i) $\{(x, y) | 3x + y = 11\}$

Write $y = 11 - 3x$ and keep only y ≥ 0.

- x = 0 → y = 11 (allowed)
- x = 1 → y = 8 (allowed)
- x = 2 → y = 5 (allowed)
- x = 3 → y = 2 (allowed)
- x = 4 → y = −1 (not allowed, negative)

So pairs:

$$\{(0, 11), (1, 8), (2, 5), (3, 2)\}$$

Domain: {0, 1, 2, 3}
Range: {11, 8, 5, 2} (often written as {2, 5, 8, 11})

(ii) $\{(x, y) | x - y = 6\}$

$$x = y + 6$$

y can be any whole number:

- y = 0 → x = 6 → (6,0)
- y = 1 → x = 7 → (7,1)
- y = 2 → x = 8 → (8,2)
- and so on...

So:

- **Domain:** all whole numbers ≥ 6 → {6, 7, 8, 9, ...}
- **Range:** all whole numbers → {0, 1, 2, 3, ...}

Question 9

$f : A \longrightarrow B$ where
$f = \{(1, 5), (2, 6), (3, 7), (4, 8)\}$,
$A = \{1, 2, 3, 4\}, B = \mathrm{N}$

- **Domain:** first components = {1, 2, 3, 4}
- **Co-domain:** B = N (set of natural numbers)
- **Range:** second components = {5, 6, 7, 8}

Values:

- $f(2) = 6$
- $f(4) = 8$

Question 10

$A = \{0, 1, 2, 3, 4\},\ B = \{2, 4, 6, 8, 10\}$

A relation from A to B is a **function** if:

- Every element of A appears exactly once as first component.

Now check each.

(i) $R_1 = \{(0, 2), (1, 4), (2, 6), (3, 8)\}$

Element 4 ∈ A is missing (no pair starting with 4).
So R_1 does **not** give an image for 4.

R_1 is not a function from A to B.

(ii) $R_2 = \{(0, 10), (1, 8), (2, 6), (2, 4), (3, 4), (4, 2)\}$

Here 2 has **two images**: 6 and 4.
This is not allowed in a function.

R_2 is not a function.

(iii) $R_3 = \{(0, 2), (1, 4), (2, 6), (3, 8), (4, 8)\}$

Every element of A = {0,1,2,3,4} appears exactly once.
So R_3 **is a function**.

- 0→2, 1→4, 2→6, 3→8, 4→8
  Two elements (3 and 4) have same image 8 → not one-one.
- Range = {2,4,6,8}, but B = {2,4,6,8,10}. 10 is not used → not onto.

So R_3 is a **many-one into function**.

(iv) $R_4 = \{(0, 2), (1, 2), (2, 2), (3, 2), (4, 2)\}$

All elements of A appear once → R_4 is a function.

- Many elements map to 2 → not one-one (many-one).
- Range = {2} only, so not onto B.

So R_4 is a **many-one into function**.

(v) $R_5 = \{(0,2),(1,4),(2,6),(3,8),(4,10)\}$

Each element of A appears once → it is a function.

- All images are different → **one-one**.
- Range = {2,4,6,8,10} = B → **onto**.

So R_5 is a **one-one correspondence (bijective function)**.

Question 11

Answer:
(i) one-one function
(ii) one-one correspondence
(iii) neither one-one nor one-one correspondence

Question 12

$P = \{a,b,c\},\ Q = \{x,y,z\},\ R = \{p,q,r,s\}$

We just need to **give examples** that satisfy the conditions.

(i) A function f from P into Q

"Into" means: function from P to Q, but **not** onto (at least one element of Q is not used).

Example:

$$f = \{(a,x),(b,y),(c,y)\}$$

- Domain P: a,b,c each appears once.
- z in Q has no preimage → f is **into**, not onto.

(ii) A function g from R onto P

Onto means: every element of P appears as an image.

Example:

$$g = \{(p, a), (q, b), (r, c), (s, a)\}$$

- Domain R: p,q,r,s each used once.
- Images: {a,b,c} = P → **onto**. (a appears twice, that's okay.)

(iii) A function h from P to R which is injective (one-one)

Injective means: different elements of P have different images.

Example:

$$h = \{(a, p), (b, q), (c, r)\}$$

- Each element of P has a different image in R → **one-one**.
- s in R is not used, but that is allowed.

(iv) A function k from Q to P which is bijective

Bijective = one-one correspondence (one-one and onto).

Example:

$$k = \{(x, a), (y, b), (z, c)\}$$

- x, y, z all used once (function).
- Images: {a, b, c} = P and all are different → one-one and onto.