بسم الله الرحمن الرحيم

Solutions of

UNIT #17

Exercise 17.3

Class 10 Math Sindh Board

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Exercise 17.3

Question 1 (statement)

Verify the commutative property of union and intersection for the following sets.
(i) $A = \{a, b, c, d, e\}$ and $B = \{a, e, i, o, u\}$.
(ii) $P = \{x \mid x \in \mathrm{Z} \wedge -3 < x < 3\}$ and $Q = \{y \mid y \in E^{+} \wedge y \leq 4\}$.

Solution 1

Definition (commutative laws). For any sets X, Y:

$$X \cup Y = Y \cup X, \qquad X \cap Y = Y \cap X.$$

(i) $A = \{a, b, c, d, e\},\ B = \{a, e, i, o, u\}$.

- $A \cup B = \{a, b, c, d, e, i, o, u\}$.
  $B \cup A = \{a, e, i, o, u, b, c, d\} = \{a, b, c, d, e, i, o, u\}$.
  So $A \cup B = B \cup A$.
- $A \cap B = \{a, e\}$ (elements common to both).
  $B \cap A = \{a, e\}$.
  So $A \cap B = B \cap A$.

(ii) First list the sets:

- $P = \{x \in \mathrm{Z} \mid -3 < x < 3\} = \{-2, -1, 0, 1, 2\}$.
- E^{+} means positive even integers, so $Q = \{2, 4\}$.
- $P \cup Q = \{-2, -1, 0, 1, 2, 4\}$.
  $Q \cup P = \{2, 4, -2, -1, 0, 1\} = \{-2, -1, 0, 1, 2, 4\}$.
  Thus $P \cup Q = Q \cup P$.
- $P \cap Q = \{2\}$.
  $Q \cap P = \{2\}$.
  Thus $P \cap Q = Q \cap P$.

So commutativity holds in all parts.

Question 2 (statement)

Verify the associative property of union and intersection for the following sets.
(i) $A = \{1, 2, 4, 5, 10, 20\},\ B = \{5, 10, 15, 20\}$ and $C = \{1, 2, 5, 10\}$.
(ii) $A = \mathrm{N},\ B = \mathrm{P}$ and $C = \mathrm{Z}$.

Solution 2

Definition (associative laws). For any sets X, Y, Z:

$$(X \cup Y) \cup Z = X \cup (Y \cup Z), \qquad (X \cap Y) \cap Z = X \cap (Y \cap Z).$$

(i) Compute unions and intersections.

- Unions:
  - $A \cup B = \{1, 2, 4, 5, 10, 15, 20\}$.
    Then $(A \cup B) \cup C = \{1, 2, 4, 5, 10, 15, 20\} \cup \{1, 2, 5, 10\} = \{1, 2, 4, 5, 10, 15, 20\}$.
  - $B \cup C = \{1, 2, 5, 10, 15, 20\}$.
    Then $A \cup (B \cup C) = \{1, 2, 4, 5, 10, 20\} \cup \{1, 2, 5, 10, 15, 20\} = \{1, 2, 4, 5, 10, 15, 20\}$.
  - Both sides equal, so associative for union holds.
- Intersections:
  - $A \cap B = \{5, 10, 20\}$. Then $(A \cap B) \cap C = \{5, 10, 20\} \cap \{1, 2, 5, 10\} = \{5, 10\}$.
  - $B \cap C = \{5, 10\}$. Then $A \cap (B \cap C) = \{1, 2, 4, 5, 10, 20\} \cap \{5, 10\} = \{5, 10\}$.
  - Both sides equal, so associative for intersection holds.

(ii) Associativity is a general law of set algebra. For example with $A = \mathrm{N}, B = \mathrm{P}, C = \mathrm{Z}$ you may check elementwise (take any element x and show membership in left side iff membership in right side). Hence both associative laws hold.

Question 3 (statement)

Verify
(a) **Distributive property of union over intersection.**
(b) **Distributive property of intersection over union**
for the following sets:
(i) $A = \{1, 2, 3, \ldots, 10\},\ B = \{2, 3, 5, 7\}$ and $C = \{1, 3, 5, 7, 9\}$.
(ii) $A = \mathrm{N},\ B = \mathrm{P}$ and $C = \mathrm{W}$.

Solution 3

Laws. For any sets X, Y, Z:

$$\text{(a)} \quad X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z), \qquad \text{(b)} \quad X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z).$$

(i) $A = \{1, \ldots, 10\},\ B = \{2, 3, 5, 7\},\ C = \{1, 3, 5, 7, 9\}$.

- First check union distributes over intersection:
  - $B \cap C = \{3, 5, 7\}$.
    Left side: $A \cup (B \cap C) = \{1, \ldots, 10\} \cup \{3, 5, 7\} = \{1, \ldots, 10\}$.
    Right side: $(A \cup B) \cap (A \cup C)$. But $A \cup B = A \cup C = A = \{1, \ldots, 10\}$, so right side is A.
    Hence equality holds.
- Now check intersection distributes over union:
  - $B \cup C = \{1, 2, 3, 5, 7, 9\}$.
    Left side: $A \cap (B \cup C) = \{1, 2, 3, 5, 7, 9\}$.

Right side: $(A \cap B) \cup (A \cap C) = \{2, 3, 5, 7\} \cup \{1, 3, 5, 7, 9\} = \{1, 2, 3, 5, 7, 9\}$.
Both sides equal, so (b) holds.

(ii) For general named sets like $A = \mathrm{N}, B = \mathrm{P}, C = \mathrm{W}$ (natural, prime, whole numbers), the distributive laws are universal; the same elementwise proofs apply.

Question 4 (statement)

Verify De Morgan's laws if $U = \{1, 2, 3, \ldots, 12\},\ A = \{1, 2, 3, 4, 6, 12\}$ **and** $B = \{2, 4, 6, 8\}$.

Solution 4

De Morgan's laws. For any sets X, Y with universe U:

$$(X \cup Y)' = X' \cap Y', \qquad (X \cap Y)' = X' \cup Y',$$

where $X' = U \setminus X$.

Given $U = \{1, \ldots, 12\}$, $A = \{1, 2, 3, 4, 6, 12\}$, $B = \{2, 4, 6, 8\}$.

- Compute complements:
  - $A' = U \setminus A = \{5, 7, 8, 9, 10, 11\}$.
  - $B' = U \setminus B = \{1, 3, 5, 7, 9, 10, 11, 12\}$.
- $A \cup B = \{1, 2, 3, 4, 6, 8, 12\}$. So $(A \cup B)' = U \setminus (A \cup B) = \{5, 7, 9, 10, 11\}$.
  $A' \cap B' = \{5, 7, 8, 9, 10, 11\} \cap \{1, 3, 5, 7, 9, 10, 11, 12\} = \{5, 7, 9, 10, 11\}$.
  Thus $(A \cup B)' = A' \cap B'$.
- $A \cap B = \{2, 4, 6\}$. So $(A \cap B)' = U \setminus \{2, 4, 6\} = \{1, 3, 5, 7, 8, 9, 10, 11, 12\}$.
  $A' \cup B' = \{5, 7, 8, 9, 10, 11\} \cup \{1, 3, 5, 7, 9, 10, 11, 12\} = \{1, 3, 5, 7, 8, 9, 10, 11, 12\}$.
  Thus $(A \cap B)' = A' \cup B'$.

So both De Morgan laws are verified.

Question 5 (statement)

If A **and** B **are subsets of** U **then prove the following by using properties.**
(i) $A \cup (A \cap B) = A$.
(ii) $A \cup B = A \cup (A' \cap B)$.
(iii) $B = (A \cap B) \cup (A' \cap B)$.
(iv) $B = A \cup (A' \cap B)$, if $A \subseteq B$.

(Here A' denotes complement of A with respect to U.)

Solution 5

We prove each identity using elementwise (membership) reasoning and known laws (absorption, distributive, etc.).

(i) $A \cup (A \cap B) = A$.

- If $x \in A$ then obviously $x \in A \cup (A \cap B)$.
- Conversely if $x \in A \cup (A \cap B)$ then $x \in A$ or $x \in A \cap B$. In both cases $x \in A$.
  Thus both sets contain the same elements, so equality holds. (This is the *absorption law*.)

(ii) $A \cup B = A \cup (A' \cap B)$.
Proof by partition of B: $B = (A \cap B) \cup (A' \cap B)$ (see (iii)). Then

$$A \cup B = A \cup ((A \cap B) \cup (A' \cap B)) = (A \cup (A \cap B)) \cup (A' \cap B).$$

By (i), $A \cup (A \cap B) = A$. So $A \cup B = A \cup (A' \cap B)$.

(iii) $B = (A \cap B) \cup (A' \cap B)$.
Every element of B either belongs to A (hence in $A \cap B$) or does not belong to A (hence in $A' \cap B$). Thus B is exactly the union of these two disjoint parts.

(iv) If $A \subseteq B$ then $B = A \cup (A' \cap B)$.
From (iii): $B = (A \cap B) \cup (A' \cap B)$. If $A \subseteq B$ then $A \cap B = A$. So $B = A \cup (A' \cap B)$. This is the same identity as (ii) but now follows directly from the subset condition.