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Solutions of

UNIT#17

Exercise 17.2

Class 10 Math Sindh Board

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Solved Exercises, Past Papers
MCQ Tests, Imprtant Formulas

Question 1

Which of the following sets are disjoint, overlapping, exhaustive and cells?

(i) $\{1, 2, 3, 5, 7\}$ and $\{4, 6, 8, 9, 10\}$

Common elements of both sets?
First set: $1, 2, 3, 5, 7$
Second set: $4, 6, 8, 9, 10$

There is **no** common element.
So, intersection is empty:

$$\{1, 2, 3, 5, 7\} \cap \{4, 6, 8, 9, 10\} = \emptyset$$

Such sets are called **disjoint sets**.

Answer: Disjoint sets.

(ii) $\{1, 2, 3, 6\}$ and $\{1, 2, 4, 8\}$

Common elements?
First set: $1, 2, 3, 6$
Second set: $1, 2, 4, 8$

Common elements are 1 and 2.
So,

$$\{1, 2, 3, 6\} \cap \{1, 2, 4, 8\} = \{1, 2\} \neq \emptyset$$

Intersection is not empty, so they are **not disjoint**.
Also their union is not a universal set here (universal set is not given).
So they are just **overlapping sets**.

Answer: Overlapping sets.

(iii) E and O when $U = \mathbb{Z}$

Here
$U = \mathbb{Z}$ = set of all integers.
E = set of all even integers.
O = set of all odd integers.

Every integer is either even or odd, so

$$E \cup O = \mathbb{Z} = U$$

Also, no integer is both even and odd at the same time, so

$$E \cap O = \emptyset$$

They are **disjoint** and also **exhaustive** (their union is the whole U).
Such sets are called **cells**.

Answer: Cells.

(iv) $A = \{0, 2, 4, \ldots\}$, $B = \mathbb{N}$, $U = \mathbb{W}$

$\mathbb{W}$ = whole numbers = $\{0, 1, 2, 3, \ldots\}$
$\mathbb{N}$ = natural numbers = $\{1, 2, 3, \ldots\}$
A = even whole numbers = $\{0, 2, 4, 6, \ldots\}$

Union:

$$A \cup B = \{0, 1, 2, 3, 4, 5, \ldots\} = \mathbb{W} = U$$

So they are **exhaustive**.

Intersection:

$$A \cap B = \{2, 4, 6, \ldots\} \neq \emptyset$$

So they are **not disjoint**, hence **not cells**.

Answer: Exhaustive sets.

(v) $\mathbb{Q}$ and $\mathbb{Q}'$ when $U = \mathbb{R}$

Here
$U = \mathbb{R}$ = set of real numbers.
$\mathbb{Q}$ = rational numbers.
$\mathbb{Q}'$ = complement of $\mathbb{Q}$ in $\mathbb{R}$ = irrational numbers.

No real number is both rational and irrational, so

$$\mathbb{Q} \cap \mathbb{Q}' = \emptyset$$

Every real number is either rational or irrational, so

$$\mathbb{Q} \cup \mathbb{Q}' = \mathbb{R} = U$$

Again they are **disjoint** and **exhaustive**, so they are **cells**.

Answer: Cells.

Question 2

If $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6, 8, 10\}$ then find:

(i) $A \cup B$

Write all elements of A and then add elements of B which are not already in A.

From A: $1, 2, 3, 4, 5, 6$
Extra from B: $8, 10$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$$

Answer: $\{1, 2, 3, 4, 5, 6, 8, 10\}$

(ii) $B \cap A$

Common elements of both sets:

Common in A and B: $2, 4, 6$

$$B \cap A = \{2, 4, 6\}$$

Answer: $\{2, 4, 6\}$

(iii) $A - B$

Elements of A which are **not** in B:

From A: $1, 2, 3, 4, 5, 6$
Remove the ones in B (2,4,6) → left with $1, 3, 5$.

$$A - B = \{1, 3, 5\}$$

Answer: $\{1, 3, 5\}$

(iv) $B - A$

Elements of B which are **not** in A:

From B: $2, 4, 6, 8, 10$
Numbers not in A are $8, 10$.

$$B - A = \{8, 10\}$$

Answer: $\{8, 10\}$

(v) $A \Delta B$ (symmetric difference)

Symmetric difference = elements which are in exactly one of the sets.

$$A \Delta B = (A - B) \cup (B - A)$$

From above: $A - B = \{1, 3, 5\}$, $B - A = \{8, 10\}$

So

$$A \Delta B = \{1, 3, 5\} \cup \{8, 10\} = \{1, 3, 5, 8, 10\}$$

Answer: $\{1, 3, 5, 8, 10\}$

Question 3

Let $U = \{1, 2, 3, \ldots, 10\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 3, 5, 7, 9\}$. Find:

(i) A'

Complement means elements in U but not in A.

From U: $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$
Remove elements of A: $1, 2, 3, 4, 5$

Left: $6, 7, 8, 9, 10$

$$A' = \{6, 7, 8, 9, 10\}$$

Answer: $\{6, 7, 8, 9, 10\}$

(ii) B'

Remove elements of B from U.

$B = \{1, 3, 5, 7, 9\}$
Remaining in U: $2, 4, 6, 8, 10$

$$B' = \{2, 4, 6, 8, 10\}$$

Answer: $\{2, 4, 6, 8, 10\}$

(iii) $A' \cup B'$

Using (i) and (ii):

$A' = \{6, 7, 8, 9, 10\}$
$B' = \{2, 4, 6, 8, 10\}$

Union: take all elements, no repetition.

$$A' \cup B' = \{2, 4, 6, 7, 8, 9, 10\}$$

Answer: $\{2, 4, 6, 7, 8, 9, 10\}$

(iv) $A' \cap B'$
Common elements in A' and B':

Common are $6, 8, 10$.

$$A' \cap B' = \{6, 8, 10\}$$

Answer: $\{6, 8, 10\}$

(v) $(A \cup B)'$
First find $A \cup B$.

$A = \{1, 2, 3, 4, 5\}$
$B = \{1, 3, 5, 7, 9\}$

Union: $\{1, 2, 3, 4, 5, 7, 9\}$

Now complement in U:

Remove $1, 2, 3, 4, 5, 7, 9$ from U.
Remaining: $6, 8, 10$.

$$(A \cup B)' = \{6, 8, 10\}$$

Answer: $\{6, 8, 10\}$

(vi) $(A \cap B)'$
First find intersection:

Common elements of A and B are $1, 3, 5$.

$$A \cap B = \{1, 3, 5\}$$

Now complement:

Remove $1, 3, 5$ from U.
Remaining: $2, 4, 6, 7, 8, 9, 10$.

$$(A \cap B)' = \{2, 4, 6, 7, 8, 9, 10\}$$

Answer: $\{2, 4, 6, 7, 8, 9, 10\}$

(vii) $A' \Delta B'$
Use symmetric difference formula:

$$A' \Delta B' = (A' - B') \cup (B' - A')$$

From above:
$A' = \{6, 7, 8, 9, 10\}$
$B' = \{2, 4, 6, 8, 10\}$

Elements in A' but not in B': $7, 9$.
Elements in B' but not in A': $2, 4$.

So

$$A' \Delta B' = \{2, 4, 7, 9\}$$

Answer: $\{2, 4, 7, 9\}$

(viii) $(A \Delta B)'$
First find $A \Delta B$.

$$A \Delta B = (A - B) \cup (B - A)$$

$A - B$: elements of A not in $B \rightarrow 2, 4$
$B - A$: elements of B not in $A \rightarrow 7, 9$

So

$$A \Delta B = \{2, 4, 7, 9\}$$

Now complement:

Remove $2, 4, 7, 9$ from U.
Remaining: $1, 3, 5, 6, 8, 10$.

$$(A \Delta B)' = \{1, 3, 5, 6, 8, 10\}$$

Answer: $\{1, 3, 5, 6, 8, 10\}$

(ix) $A - B'$

We know
$A = \{1, 2, 3, 4, 5\}$
$B' = \{2, 4, 6, 8, 10\}$

Elements of A that are **not** in B':

From A, remove 2 and 4 → left with $1, 3, 5$.

$$A - B' = \{1, 3, 5\}$$

Answer: $\{1, 3, 5\}$

(x) $A' - B$
$A' = \{6, 7, 8, 9, 10\}$
$B = \{1, 3, 5, 7, 9\}$

Elements of A' that are **not** in B:

Remove 7 and 9 → left with $6, 8, 10$.

$$A' - B = \{6, 8, 10\}$$

Answer: $\{6, 8, 10\}$

Question 4

$U = \{x \mid x \in Z, -4 < x < 6\}, P, Q \subseteq U$. Show that:

> (i) $P - Q = P \cap Q'$

Let $x \in P - Q$.
Then $x \in P$ and $x \notin Q$.
"Not in Q" means $x \in Q'$.
So $x \in P$ **and** $x \in Q' \rightarrow x \in P \cap Q'$.

So every element of $P - Q$ is in $P \cap Q'$.
Similarly, if $x \in P \cap Q'$, then $x \in P$ and $x \notin Q$.
So $x \in P - Q$.

Thus $P - Q = P \cap Q'$.

> (ii) $Q - P = Q \cap P'$

Same idea.

Take $x \in Q - P$.
Then $x \in Q$ and $x \notin P$, so $x \in P'$.
Therefore $x \in Q \cap P'$.

Conversely, if $x \in Q \cap P'$, then $x \in Q$ and $x \notin P$,
so $x \in Q - P$.

Hence $Q - P = Q \cap P'$.

> (iii) $(P \cup Q)' = P' \cap Q'$

This is De Morgan's law.

Let $x \in (P \cup Q)'$.
Then $x \notin P \cup Q$.
So x is not in P and not in Q.
Therefore $x \in P'$ and $x \in Q'$, so $x \in P' \cap Q'$.

Conversely, if $x \in P' \cap Q'$ then x is not in P and not in Q.
So $x \notin P \cup Q$, which means $x \in (P \cup Q)'$.

Thus $(P \cup Q)' = P' \cap Q'$.

> (iv) $(P \cap Q)' = P' \cup Q'$

Again De Morgan's law.

Let $x \in (P \cap Q)'$.
Then $x \notin P \cap Q$.
So x is **not in both together**.
That means $x \notin P$ **or** $x \notin Q$.
So $x \in P'$ or $x \in Q'$.
Therefore $x \in P' \cup Q'$.

Conversely, if $x \in P' \cup Q'$, then x is not in at least one of the sets P or Q.
So x cannot be in both at the same time, meaning $x \notin P \cap Q$.
So $x \in (P \cap Q)'$.

Hence $(P \cap Q)' = P' \cup Q'$.

Question 5

If $A = \{2n \mid n \in \mathrm{N}\}$, $B = \{3n \mid n \in \mathrm{N}\}$, $C = \{4n \mid n \in \mathrm{N}\}$ then find:

Remember:

- $A = \{2, 4, 6, 8, 10, \dots\}$ (multiples of 2)
- $B = \{3, 6, 9, 12, 15, \dots\}$ (multiples of 3)
- $C = \{4, 8, 12, 16, \dots\}$ (multiples of 4)

(i) $A \cap B$

Common elements are numbers which are multiples of both 2 and 3 → multiples of 6.

So

$$A \cap B = \{6, 12, 18, \dots\} = \{6n \mid n \in \mathrm{N}\}$$

Answer: $\{6n \mid n \in \mathrm{N}\}$

(ii) $A \cup C$

Every element of C is a multiple of 4, and every multiple of 4 is also a multiple of 2 ($4n = 2(2n)$), so $C \subset A$.

Therefore, union is just A:

$$A \cup C = A = \{2n \mid n \in \mathrm{N}\}$$

Answer: $\{2n \mid n \in \mathrm{N}\}$

(iii) $B \cap C$

Common elements are numbers which are multiples of both 3 and 4 → multiples of 12.

$$B \cap C = \{12, 24, 36, \dots\} = \{12n \mid n \in \mathrm{N}\}$$

Answer: $\{12n \mid n \in \mathrm{N}\}$

(iv) $A \cap C$

Common elements are multiples of both 2 and 4 → actually multiples of 4 (since every multiple of 4 is automatically multiple of 2).

So

$$A \cap C = C = \{4n \mid n \in \mathrm{N}\}$$

Answer: $\{4n \mid n \in \mathrm{N}\}$

Question 6

Given $A_n = \{n, n+1, n+2, \dots\}$, for all $n \in \mathrm{N}$. Find:

First write some terms:

- $A_3 = \{3, 4, 5, 6, 7, \dots\}$
- $A_5 = \{5, 6, 7, 8, 9, \dots\}$
- $A_7 = \{7, 8, 9, 10, 11, \dots\}$
- $A_{11} = \{11, 12, 13, \dots\}$
- $A_{13} = \{13, 14, 15, \dots\}$
- $A_{15} = \{15, 16, 17, \dots\}$
- $A_8 = \{8, 9, 10, 11, \dots\}$
- $A_9 = \{9, 10, 11, 12, \dots\}$

(i) $A_3 \cup A_5$

A_3 starts from 3, A_5 starts from 5.
All elements of A_5 are already inside A_3.

So

$$A_3 \cup A_5 = A_3$$

Answer: A_3.

(ii) $A_7 \cap A_{11}$

A_7 starts from 7, A_{11} starts from 11.
Common elements will start from 11 onwards.

So intersection is the set that starts from the bigger starting number, i.e. 11:

$$A_7 \cap A_{11} = A_{11}$$

Answer: A_{11}.

(iii) $A_{15} - A_{13}$

$A_{13} = \{13, 14, 15, 16, 17, \dots\}$
$A_{15} = \{15, 16, 17, 18, \dots\}$

Every element of A_{15} is also in A_{13} (because A_{13} already contains 15,16,17,...).
So there is **no** element which is in A_{15} but not in A_{13}.

Therefore

$$A_{15} - A_{13} = \emptyset$$

Answer: ∅ (empty set).

(iv) $A_9 \Delta A_8$
Write first few elements:

$A_8 = \{8, 9, 10, 11, 12, \dots\}$
$A_9 = \{9, 10, 11, 12, \dots\}$

Elements in exactly one of them:

- 8 is only in A_8, not in A_9.
- 9,10,11,... are in both.

So symmetric difference is:

$$A_9 \Delta A_8 = \{8\}$$

Answer: $\{8\}$